ADVERTISING STRATEGIES FOR NEW PRODUCT DIFFUSION IN A REGULATED MARKET: PROPOSITIONS AND NORMATIVE RECOMMENDATIONS

By

ARINDAM DUTTA

DEPARTMENT OF INDUSTRIAL AND MANAGEMENT ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY KANPUR
May, 2005

ADVERTISING STRATEGIES FOR NEW PRODUCT DIFFUSION IN A REGULATED MARKET: PROPOSITIONS AND NORMATIVE RECOMMENDATIONS

A Thesis submitted

In Partial Fulfillment of the Requirements
for the Degree of

MASTER OF TECHNOLOGY

May, 2005

By

ARINDAM DUTTA

DEPARTMENT OF INDUSTRIAL AND MANAGEMENT ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY
KANPUR – 208016 (INDIA)

CERTIFICATE

It is certified that the work contained in the thesis entitled "*Advertising Strategies for New Product Diffusion in a Regulated Market: Propositions and Normative Recommendations*" by Arindam Dutta (Roll No. Y3114004) has been carried out under my supervision and that this work has not been submitted elsewhere for degree.

Dr. Sanjeev Swami
Assistant Professor
Industrial & Management Engineering
Indian Institute of Technology, Kanpur

May 2005

ACKNOWLEDGEMENT

I wish to express my deep sense of gratitude and indebtedness to my thesis supervisor, Dr. Sanjeev Swami. Without his consistent help and guidance it would have been difficult to accomplish this research.

I am very much thankful to Dr. R.R.K. Sharma, Dr. A.K. Mittal, and Dr. R.N. Sengupta for their support to me. I would like to thank all my friends at IITK for making my stay at IITK very enjoyable.

I wish to thank IME staff for extending their assistance. I thank all those who have contributed directly or indirectly to my thesis.

May 2005

Arindam Dutta

ABSTRACT

Global companies are turning to newly emerging markets for business expansion due to the saturation of the developed markets of the world. These emerging markets offer a significant and growing number of buyers and are increasingly becoming attractive because of the potential of immediate sales. Pioneering firms, which are first to launch a new product in this market will gain foothold and gradually their sales will increase over time. It is therefore imperative that product diffusion models be developed for this market situation to explain the spread of a new product among a set of prospective adopters over time.

The objective of this thesis is to address a problem which is relevant to a global corporation decision maker who is developing advertising strategies under the following situation. The decision maker has a new product for which demand exists in a regulated market. However, since the market has not been opened up, the primary channels for distribution do not exist for consumers in the market. Therefore, the consumers are compelled to rely on secondary channels such as acquaintances residing in open markets abroad. A modeling framework is developed for this purpose which characterizes the product diffusion patterns, both before and after opening of the market. Further, optimal normative policies for advertising are derived by applying optimal control technique to the proposed diffusion model. The effect of various model parameters on the diffusion model as well as on the optimal advertising policy is also examined.

CONTENTS

List of Figures

List of Tables

Chapter 1
Introduction

Economic liberalization and international trade agreements have led to loosening of restrictions on commercial activities. As the developed markets of the world are getting saturated, companies are turning to newly emerging markets for business expansion. The other reasons behind the attractiveness of emerging markets are the potential of immediate sales and rising strength of these economies. Pioneering firms, which are first to launch a new product in this market will gain foothold and gradually their sales will increase over time, even if they maintain their original shares due to the overall growth of the market. These rapidly developing economies offer a significant and growing number of buyers. Thus, the first movers will benefit from improving economies of scale as production rises to meet demand (Nakata and Sivakumar 1997).

In the initial stages of trade liberalization in a developing country like India, which has initiated the liberalization process quite late, early entry by a foreign firm is favorably looked on as the firm's commitment to the emerging market. It therefore enjoys fast and easy access to the political decision-makers and key bureaucrats and can develop personal relationship with them. Such relationship is considered as essential to do business in an emerging market and it offers several benefits (Khanna and Palepu 1997). Also, an economic advantage comes in the form of lower product and marketing communications adaptation costs. Products originally designed for developed markets and later introduced to emerging markets require little to no adaptation if targeted to affluent urbanites (Hill 1984). These consumers have the financial means to afford costly foreign products and services and may find other channels of adoption if they are not available locally. However, this is truer of products less sensitive to cultural preferences, such as durables or electronic goods.

1.1 Globalization and Regionalization

In the past two decades, there has been increasing globalization of production and markets in the world economy. Trade and investment are complements, rather than substitutes, today. However, when multinational enterprises (MNEs) seek to locate their affiliates in different countries and sell to globalized markets, trade protection is an impediment to the inflow of FDI. In this section we review the globalization and regionalization trends in the world and the Asia-Pacific as described by Sazanami (1996).

The emergence of local markets responding to the rapid economic growth and the rise in the standard of living is an important factor in attracting FDI. Japanese FDI abroad started to show a significant increase from the early 1980s; when Japanese MNEs found it more attractive to build factories abroad than to invest in the stagnant domestic market. Rapid appreciation of the yen after 1985 pushed Japanese MNEs to seek lower-cost production locations abroad. Appreciation of the yen against the dollar after 1993 necessitated Japanese MNEs to further expand offshore production in low-wage countries. Also, the deregulation in Asian countries to attract foreign firms helped to accelerate the relocation of Japanese MNEs. Japanese firms have been leading investors in emerging economies, particularly those in Asia (Belderbos and Sleuwaegen 1996). Japanese MNEs started to invest in East Asia, not only for advantageous offshore production conditions, but also for the sales in the local and regional markets. In 1994, East Asia received the largest amount of manufacturing FDI (US$4,940 million) from Japan, substantially larger than Japanese manufacturing FDI to North America (US$4,763 million) in the same year.

1.2 Emerging Markets

The term "emerging market" was first coined in 1981 by Antoine W. van Agtmael, an employee of the World Bank's International Finance Corporation. Although there is no universally accepted definition, normally three tests are applied to categorize a developing economy as an emerging market: absolute level of economic growth characterized by per capita GDP, economic growth rate as defined by annual GDP growth rate and the extent to which marketing infrastructure of a country supports a free market system (Arnold and Quelch 1998). Rahman and Bhattacharyya (2003) propose that the definition of an emerging market should be based on the following attributes. First, it should hold out the promise of substantial economic growth in the future. Second, the economy was opened in the recent past for direct foreign investment and the trade liberalization process would continue in the future. Third, it has an institutional infrastructure, which facilitates marketing transactions.

Emerging markets merit special attention from the business point of view. Although advanced economies account for more than three fourths of the global market, many of them suffer from economic recession or stagnation, ageing population and low fertility rates leading to low growth in various product markets (Nakata and Sivakumar 1997). On the other hand, emerging markets represent some of the fastest growing economies.

Vilfredo Pareto, a nineteenth century economist, observed that a small proportion of population accounts for a disproportionately large amount of national income that the same

observation still applies to many emerging markets. The wealthy urbanites are endowed with the necessary education and consumption experiences as well as the means to afford costly foreign goods and services (Nakata and Sivakumar 1997). The emerging economies therefore constitute additional emerging markets of goods and service of global corporations.

1.3 Market Pioneering

Market pioneering, an approach in which a firm is first to offer a distinctively new product to the market is a commonly recognized form of corporate entrepreneurship. Market pioneering is commonly identified as one potential manifestation of entrepreneurial behavior (Miller 1983). Market pioneering represents an organization which proactively creates or is among the first to enter a new product-market arena that others have not recognized or actively sought to exploit. By engaging in pioneering the firm, in essence, takes the competition to a new arena where its first or early mover status is hoped to create some basis for sustainable competitive advantage. For example, the pioneering firm may be able to create the industry standard or define the benchmark against which later entrants are judged. Thus, firms that engage in pioneering are entrepreneurial in that they exploit market opportunities in a preemptive fashion, redefining where and how the competitive game is played in the process.

1.4 First Mover Advantages

Closely related to the concept of pioneer, a first mover is defined as a firm that is first to produce a new product, uses a new process or enters a new market. According to the findings of Rahman and Bhattacharyya (2003), it pays to be a first mover in an emerging market. When a firm is first to enter a market, it has certain advantages over other firms who are late entrants, because it is able to influence and has the ability to create and capture value in the new market. A large number of papers discussing the relative advantages of a first mover have been published in management literature. These papers suggest that though a first mover may not benefit in form of spectacular gains, it certainly enjoys several competitive advantages (Robinson and Fornell 1985, Robinson 1998, Rogers 1983). Mittal and Swami (2004) further substantiate these results in an Indian setting.

A first mover enjoys a differentiation advantage because its customers are not distracted by competing choices. It therefore attracts more attention of consumers than a new entrant. In the early stages of market evolution, consumers have little knowledge about the ideal product attributes and their ideal combination. A first mover can therefore shape

consumer's perception in these respects to its advantages. It can even become the category standard against which consumers compare the offers of late entrants while making a purchase decision. A first mover also attains critical sales volumes and accumulates valuable research and development and market experience before any other competitor. It therefore benefits from cost advantages associated with scale and experience economies. It can use these cost advantages either to achieve higher margins or lower product prices to discourage competitors from entering the market. Besides, a first mover can secure patents and pre-empt domination of distribution and communication channels.

1.5 Product Innovativeness

In the literature, innovativeness typically is distinguished either by technology newness and market newness or by whether the product is new to the firm or new to the world. We review the impact of product innovativeness as done by Lee and O'Connor (2003).

The issue of identifying factors that account for new product success and failure has drawn substantial attention. Among those many factors, product innovativeness is one of the most important. Some of the literature work argues that product innovativeness positively impacts new product performance because it increases a firm's competitive advantage which in turn creates additional incentives for firms to invest in innovations and increase product innovativeness as they attempt to compete in high-tech markets. Other studies indicate that innovativeness negatively impacts performance because of customers' fears associated with adopting unproven technology. These studies emphasize the negative effects that increased product innovativeness may have on the uncertainty that consumers experience (that is, high switching costs, high risk, and increased investment of time to learn new behaviors) with highly innovative products. Technological fear is an essential issue for commercialization of high-tech products. Some consumers deal with the fear of uncertainty and new learning requirements of the new innovative products simply by avoiding or hesitating in purchases of the new and improved version.

Innovativeness itself does not guarantee success. A successful innovation must be novel and, at the same time, easy to comprehend. Without an appropriate introduction strategy, a product's innovativeness may be perceived by customers as offering uncertainty and risk rather than as providing superior benefits.

1.6 Product Launch Strategy

Communication with customers to manage their perceptions of product innovativeness is critically important, especially when launching a highly innovative product that customers may reject due to lack of product knowledge. Several empirical studies have investigated successful launch strategies under varying levels of innovativeness. The literature on launch strategy measures innovativeness primarily based on product newness to the firm and affords much less attention to the impact of product newness on consumption behaviors. The impact of innovativeness on consumption behaviors depends on how customers perceive the new product. If an innovation is perceived as very difficult to adopt rather than as a unique product with substantial new benefits, the marketer's challenge is to work hard in communicating with customers to translate the technical uncertainty into useful benefits. Appropriate communication strategies at launch can reduce effectively the negative impact of technical fears on the adoption of an innovative product. Here we discuss nature of the communication messages for effective management of customers' perceptions of product innovativeness as described by Lee and O'Connor (2003).

A firm's communication strategy is a critical element of its launch plan—the element most directly responsible for aiding the market's acceptance of a new product. A lack of product knowledge causes a need for customers to be educated before the innovative product is introduced to the market. The two commonly accepted elements of a communication strategy are: preannouncement strategy and advertising strategy. Preannouncement not only preempts the market against competitors but also familiarizes potential customers with the new product concept and help shape their expectations. Second, advertising strategy plays an essential role throughout the purchasing decision process. Empirical results from previous studies indicate that the magnitude of advertising expenditures (or advertising effort) significantly impacts new product. Beyond the level of expenditures, the content of the advertising message is critical to help ease customer anxiety about new technologies.

1.7 Current Problem

The objective of this thesis is to address a problem which is relevant to a global corporation decision maker who is developing advertising strategies under the following situation. The decision maker has a new product for which demand exists in a regulated market. However, since the market has not been opened up, the primary channels for distribution do not exist for consumers in the market. Therefore, the consumers are compelled to rely on secondary channels such as acquaintances residing in open markets abroad. The

current research seeks to study the product diffusion process which will take place under the market situation described above. A modeling framework is developed for this purpose which characterizes the product diffusion patterns, both before and after opening of the market. The effect of various model parameters on the diffusion process is studied. Further, optimal normative policies for advertising are derived by applying optimal control technique to the proposed diffusion model. The normative recommendations proposed in this research seek to answer various questions of interest to the decision maker. Like, does it pay to advertise for the product before the market opens? Should there be any difference in advertising levels before and after the market opens? What should be the general trend of the advertising expenditure rate for the whole planning horizon? Further, the effect of various model parameters on the optimal advertising policies is also studied.

The above problem has not been studied in the literature so far, but has practical relevance for the kind of situations faced by Japanese electronics goods manufacturers advertising in Indian market before deregulation (1991). The global corporations which are targeting the market of emerging economies likely to face a similar situation will gain valuable insights from this research. The model can also be applied to a situation faced by a company which has regional presence in one part of a country, and can obtain access to another part only after some time.

The rest of the thesis is organized as follows. First, basic models of new product diffusion are reviewed from the literature in Chapter 2, which will help in developing a model which is appropriate for the particular situation addressed by this research. In chapter 3, the problem is formulated in detail and the diffusion model is developed accordingly. The proposed model parameters are studied in Chapter 4 by doing numerical analysis of the proposed model. Then in Chapter 5, optimal control technique is applied to the diffusion model and based on the results obtained, theorems and propositions are derived for optimal advertising policy. The effect of model properties is studied in Chapter 6, comparing the various advertising policies by doing numerical analysis. Managerial implications are discussed in Chapter 7, where all the results obtained on optimal advertising policies are summarized and normative recommendations for advertising policy is given. Finally, the thesis is concluded in Chapter 8, and limitations of the current research and directions for future research is discussed.

Chapter 2
Literature Review

Diffusion of innovations is the process by which an innovation is adopted by a society over time (Rogers and Shoemaker 1971). Since its introduction to marketing in the 1960s, innovation diffusion theory has sparked considerable research among consumer behavior, marketing management, and management and marketing science scholars. Researchers in consumer behavior have been concerned with evaluating the applicability of hypotheses developed in the general diffusion area to consumer research. The marketing management literature has focused on the implications of these hypotheses for targeting new product prospects and for developing marketing strategies aimed at potential adopters. Researchers in management and marketing science have contributed to the development of diffusion theory by suggesting analytical models for describing and forecasting the diffusion of an innovation in a social system. More recently, this literature has also been concerned with developing normative guidelines for how an innovation should be diffused in a social system (Mahajan, Muller, and Bass 1990).

Mahajan and Muller (1979) have stated that the objective of a diffusion model is to present the level of spread of an innovation among a given set of potential adopters. The purpose of the diffusion model is to depict the successive increases in the number of adopters and predict the continued development of a diffusion process already in progress. In the product innovation context, diffusion models focus on the development of a life cycle curve and serve the purpose of forecasting first-purchase sales of innovations.

2.1 The Adoption Process

The adoption process includes the steps an individual goes through from the time he hears about an innovation until final adoption. A widely accepted model of the diffusion of innovations is that of Rogers (1983). This model defines a process by which individual adopt a new innovation. Each member of the social system faces his/her own innovation-decision that follows a 5-step process:

1) *Knowledge* – person becomes aware of an innovation and has some idea of how it functions,
2) *Persuasion* – person forms a favorable or unfavorable attitude toward the innovation,

3) *Decision* – person engages in activities that lead to a choice to adopt or reject the innovation,
4) *Implementation* – person puts an innovation into use,
5) *Confirmation* – person evaluates the results of an innovation-decision already made.

According to Rogers (1983), individual must first become aware of the innovation. Once awareness of an innovation is established, an individual can at any point enter a persuasion stage during which the individual seeks and processes information in order to decide whether to adopt the innovation. The timing of the active portion of this stage is highly dependent on the individual and the context in which the he is operating. At several points in time, the individual may make a decision not to adopt, to postpone adoption, to continue the search for information, or to adopt the new innovation. This persuasion stage is followed by an implementation stage in which the actor enacts the decision. Finally, individuals reevaluate or confirm their decisions to adopt and / or their implementation of the decision. The result may be either continuance or discontinuance of the adoption.

The innovation decision is made through a cost-benefit analysis, in which the major obstacle is uncertainty. People will adopt an innovation if they believe that it will, all things considered, enhance their utility. Since people are on average risk-averse, the uncertainty will often result in a postponement of the decision until further evidence can be gathered. Each individual's innovation decision is largely framed by personal characteristics, and this diversity is what makes diffusion possible. For a successful innovation, the adopter distributions follow a bell-shaped curve, the derivative of the S-shaped diffusion curve, over time and approach normality.

Types of Adopters

The difference among individuals in their response to new ideas is called their innovativeness; it represents the degree to which an individual is relatively or late in adopting a new product or idea. Individuals are often classified into different adopter categories on the basis of their innovativeness, as illustrated in Figure 2.1.

Diffusion scholars divide this bell-shaped curve to characterize five categories of member innovativeness, where innovativeness is defined as the degree to which an individual is relatively earlier in adopting new ideas than other members of a system. These groups are: 1) innovators, 2) early adopters, 3) early majority, 4) late majority, and 5) laggards.

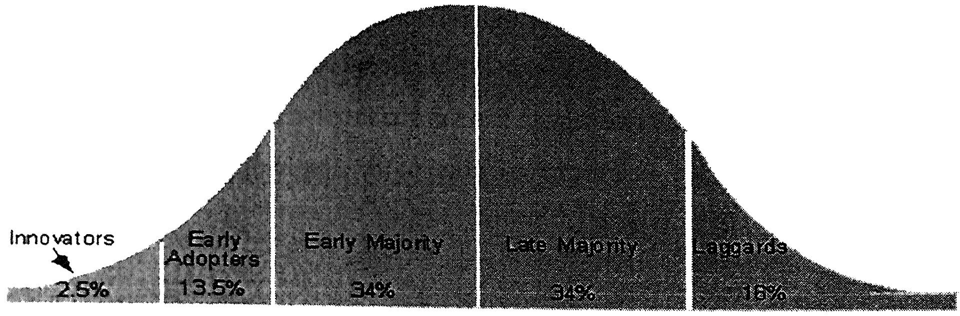


FIGURE 2.1. Categories of Adopters.

2.2 Hazard-Rate Models

Hazard rates, which study the probability of a population member performing a certain behavior, are based on research of time to failure in statistics (Kalbfleisch and Prentice 1980). There are several ways to describe the timing of when an event will take place: at any time we can look at the survivor function, $Q(t)$; the failure-time distribution, $F(t)$; the probability density function, $f(t)$; or the hazard function, $h(t)$.

The hazard function is specified as follows:

$$h(t) = \lim_{\delta t \to 0} \frac{[\Pr(Event\_occurs\_in(t, t+\delta t) \mid T \geq t)]}{\delta t} \tag{2.1}$$

where T is the time that the event occurs. It is the conditional probability of an event occurring in period, $(t, t+\delta t)$, given that it has not occurred to date.

The hazard function may be expressed in terms of the other variables by noting that

$$h(t) = \frac{f(t)}{Q(t)} = \frac{f(t)}{1 - F(t)} \tag{2.2}$$

2.3 Aggregate Diffusion Models

The early models of first-purchase diffusion of new product in marketing which attempted to describe the penetration and saturation aspects of the diffusion process were those of Fourt and Woodlock (1960), Mansfield (1961), and Bass (1969). Prior to 1969, most diffusion models in marketing could be classified as pure innovative or pure imitative. A pure innovative model assumes that only innovative or external influences are operative in the

diffusion process while a pure imitative model assumes that the only effects on the process are driven by imitation or word of mouth.

Fourt and Woodlock (1960) assumed that the diffusion process is driven primarily by the mass-media communication or the external influence; hence it is a pure innovative model. Whereas, the model proposed by Mansfield (1961) assumes this process is driven by word of mouth and hence is a pure imitative model. But the quantitative modeling of innovation diffusion in marketing started with the introduction of the simple diffusion model by Bass (1969).

2.3.1 Bass Model (1969)

According to this model, once an innovation is introduced, it spreads through the population, in which non adopters are influenced by adopters as they contact them, leading to an eventual adoption. The Bass model assumes that potential adopters of an innovation are influenced by two means of communication - mass media and word of mouth. Further it assumes that the potential adopter population can be divided in to two groups, one influenced only by mass-media communication (external influence) and the other by word-of-mouth communication (internal influence). Bass (1969) termed the first group "Innovators" and the second group "Imitators".

The Bass model derives from a hazard function (the probability that an adoption will occur at time t given that it has not yet occurred). The density function of time to adoption is given by $f(t)$ and the cumulative function of adopters at time t is given by $F(t)$.

According to Bass diffusion model, new product demand $S(X)$ is a function of the number of adopters (X),

$$S(X) = \frac{dX}{dt} = (a + bX)(N - X) \tag{2.3}$$

where $X(t)$ is the cumulative adoptions up to but not including t, dX / dt is the adoptions in time t, N is the market potential. Bass (1969) referred to the parameter a as the "coefficient of innovation" and the parameter b as the "coefficient of imitation". A plot of $X(t)$ depicts a symmetrical S curve as shown in Figure 2.2.

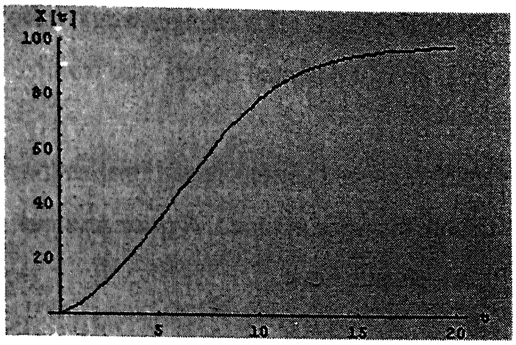


FIGURE 2.2. Bass Diffusion Curve.

By solving the differential equation, *X(t)* can be expressed as a function of *t* as follows:

$$X(t) = \frac{1 - e^{-(a+b)t}}{1 + \frac{b}{a} e^{-(a+b)t}} \tag{2.4}$$

The S-shaped solution provides excellent empirical fit for the timing of initial purchase for wide range of consumer durables.

Many diffusion models have been developed to investigate the aggregate adoption pattern of an innovation over its life cycle. While applications of the Bass model and its variants lie mainly in sales forecasting, new product development, and marketing strategy formulation, use of diffusion models for descriptive purpose is increasing (Mahajan, Muller, and Bass 1990). Since the original Bass model is limited, many extensions and variants have been introduced.

2.3.2 Bass Model Extensions

Since the publication of the Bass model, research on modeling of the diffusion of innovations in marketing has resulted in an extensive literature. Contributions of this literature through the 1970s were reviewed by Mahajan, Muller, and Bass (1990).

2.3.2.1 Models incorporating Price

Product price plays an important role in the diffusion process. However there is no consensus among diffusion researchers on how price should be specified on the diffusion model. Some authors (Robinson and Lakhani 1975, Bass, 1980, Dolan and Jeuland, 1981) argue that a price decrease would engender an increase in the probability of adoption. Hence a price reduction would only motivate potential buyers to make an earlier adoption, with no

impact on the total market for the product. On the contrary, other researchers (Mahajan and Peterson 1978, Horsky and Simon 1983) assert that a lower price would place the product within the budget limitations of a greater number of buyers, thus affecting the market potential for the product. Under this view. price changes would have a long-term impact, by increasing/decreasing the total pool of potential buyers who would eventually buy the product.

Mesak and Berg (1995) reviewed various operationalizations of price incorporation in diffusion equation. Let *P* denote the price at time *t.* A price response for first purchases (without repeat sales) *h(P)*, are given below:

$$S = (a + bX)(N - X) * h(P) \tag{2.5}$$

According to (2.5), price affects both the parameters a and b equally. This is accomplished by multiplying the Bass model by *h(P)*. This implies that the market potential N is reached more/less quickly when price decreases/increases.

$$S = (a + bX)(N * h(P) - X) \tag{2.6}$$

Equation (2.6) implies that price affects market potential. The total number of potential adopters in the market increases/decreases when product adoption is within the budgetary limitations of more/fewer individuals.

$$S = (a * h(P) + bX)(N - X) \tag{2.7}$$

In (2.7) price affects parameter a only. One can conceive this type of effect when a large portion of opinion leaders adopt the product as a function of price.

$$S = (a + bX * h(P))(N - X) \tag{2.8}$$

In (2.8), price affects parameter b only. One can imagine that (2.8) may illustrate situation in which price changes enhance information-seeking/transmitting behavior. For example, a change in price may prompt communications between adopters and non adopters.

Thus, price can be incorporated as a separable function or non separable function of the diffusion process, or as affecting the market potential (Dockner and Jorgensen 1988, Kalish 1983).

Robinson and Lakhani (1975) introduced price in a multiplicative way as follows:

$$\dot{X} = (N - X)[a + bX]e^{-dP(t)} \tag{2.9}$$

where $P(t)$ is price and d is a price sensitivity parameter.

An alternative formulation was introduced by Mahajan and Peterson (1978, 1982). Here the potential population is a function of price, that is, $N = N(P)$ which leads to the following model:

$$\dot{X} = [N(P) - X][a + bX] \tag{2.10}$$

where $N(P) = N_o e^{-dP}$.

The interpretation of the model is that the size of the potential population is a function of price, while the conditional adoption rate still depends linearly on penetration level.

2.3.2.2 Models incorporating Uncertainty

Diffusion of the innovation may be delayed by consumer's uncertainties about the performance of the product. The consumers that are sufficiently risk-averse will delay adoption and will wait for information generated by early adopters. Uncertainty about the performance may be due to lack of information about the effectiveness or quality of the product. Some diffusion models in literature have incorporated uncertainty about product performance. Bernhardt and MacKenzie (1972) postulate that potential adopters apply 'safety margins' to their estimates of the value of adoption. Value net of this safety margin must exceed pre-adoption value to induce adoption. Kalish (1983) developed a model in which awareness and adoption of the innovation is treated as separate steps, each dependent on current uncertainty and product attributes. Potential adopters are those whose risk-adjusted valuation of the innovation exceeds its price; the discount due to risk-aversion declines with the number of adopters. Gilbert and Train (1981) propose a model in which consumers have identical uncertainties but vary in risk-aversion.

Oren and Schwartz (1988) propose a model similar to that of Gilbert and Train (1981) in that uncertainties are reduced according to a Bayesian learning model. Customers have varying risk aversion, and are modeled as expected utility maximizers. Rate of adoptions is related to rate of change in underlying uncertainty and potential adopters update on the actual results obtained by the adopters.

If the population is expressed as a cumulative distribution frequency $F(\rho^*)$ over their risk-aversion parameter (ρ) such that the fraction of the total population for whom their risk-aversion given by ρ is less than the risk-aversion threshold (ρ^*), then the fraction of the population who will go ahead with their decision to purchase the product will equal to $F(\rho^*)$.

Let λ be rate at which number of customers makes purchase decision then the rate at which the product will be purchased assuming single unit purchased by a customer, is given as follows:

$$\dot{X}(t) = \lambda F(\rho^*) \tag{2.11}$$

If an exponential distribution on risk aversion is assumed on the population then

$$F(\rho) = 1 - e^{-\gamma\rho}; 0 \leq \rho \leq \infty \tag{2.12}$$

where γ is a shape parameter which describes a particular shape of the exponential distribution and is equal to the reciprocal of the mean risk aversion in the population.

Since all consumers with risk aversion less than this threshold prefer the new product, a larger threshold implies greater market share for the product. Oren and Schwartz (1988) have shown how threshold changes over time as uncertainty declines and have derived an expression for the risk-aversion threshold (ρ^*),

$$\rho^* = \kappa(X + X_o + 1) \tag{2.13}$$

where X and X_o are the cumulative adoptions and prior adoptions respectively and κ is a constant parameter. On substituting the expression for ρ^* in (2.11),

$$\dot{X} = \lambda[1 - e^{-\gamma\kappa(X + X_o + 1)}] \tag{2.14}$$

Cumulative purchases increases so long some portion of the population distribution falls below the current risk aversion threshold.

2.3.2.3 Models incorporating Quality

Teng and Thompson (1993) have dealt with the optimal price and quality policies for the introduction of a new product. On the supply side, the firm wants to determine the unit price and quality level over time given that unit cost declines along a learning curve, and increases if quality is made greater. On the demand side, dynamic demand is related to price and quality, as well as to the cumulative sales. Sales at time t is a function of price, denoted by $p(t)$, and quality level $q(t)$, as well as the cumulative sales $x(t)$. That is, the sales rate function

$$s = f[p(t), q(t), x(t)] \tag{2.15}$$

The demand growth function is assumed to be twice differentiable and increases with quality and decreases with price. Therefore,

$$\frac{\partial s}{\partial p} < 0, \frac{\partial s}{\partial q} > 0 \text{ and } \frac{\partial^2 s}{\partial p \partial q} = \frac{\partial^2 s}{\partial q \partial s}$$

Specific cases were considered, firstly where sales depend on price and quality, but not on word of mouth and also the case where demand functions are separable.

(1) $f(p,q,x) = h(q,x)P(p)$, (2.16)

(2) $f(p,q,x) = k(p,q)X(x)$, (2.17)

(3) $f(p,q,x) = w(p,x)Q(q)$ (2.18)

A particular form for $h(q,x)$ in (2.16) is given as follows:

$$h(q,x) = (e_1 + e_2 q)(M - x) + (e_3 + e_4 q)(M - x)x$$

= (Quality effects on innovators) + (Quality effects on imitators).

$$= (M - x)(e_1 + e_2 q + e_3 x + e_4 qx) \tag{2.19}$$

2.3.3 Kalish Model (1985)

Kalish integrated price, advertising, and uncertainty in a single model. The framework suggested by Kalish (1985) that the actual adoption depends on price and individual's valuation of the product. The adoption of a new product is characterized by two steps: awareness and adoption. The rate of adoption is determined by awareness diffusion, which is controlled by advertising, and the rate of growth of the potential adopter population, which is controlled by price.

Awareness diffusion: Information about the product is spread by advertising and word of mouth, by those who are aware but who have not adopted and by those who have adopted the innovation. The awareness diffusion equation is given as:

$$\dot{I} = [1 - I]\left[f(A) + bI + b'\left(\frac{X}{N_o}\right)\right] \tag{2.20}$$

Here N_o is the relevant population size, $I(t)$ is the proportion of the population aware by the time t, $X(t)$ is the number of adopters by t, b and b' are parameters.

Information diffusion is assumed to be homogeneous. Unaware individuals can become aware by word of mouth, or by advertising. Thus the conditional likelihood of becoming aware is proportional to the number of "transmitters", and to the advertising effectiveness.

Here $A(t)$ is the advertising spending rate and $f(A)$ is the likelihood that a randomly chosen individual is exposed to advertising.

Market potential: Individuals value products differently. A customer will buy a product if its price of the product is less than its value to him. Individuals are also assumed to be risk-averse and, therefore more experience the market has with a product, the less uncertain the valuation of the product is to the population and hence, the greater the market potential.

The risk adjusted market potential at price (P) and discount (u) is $N(P/u)$ where u can be written as a function of penetration as follows:

$$u = u(X / N_o)$$

Here $u(X/N_o)$ is defined as the ratio of the value of the uncertain product to the value under certainty. It is assumed that the level of uncertainty in the market decreases as the number of adopters X increases, and that u is the same across all customers at a given point in time.

Dynamics of adoption: A constant proportion of the aware population that has not adopted the product, will adopt at a rate given by the adoption equation

The aggregate adoption equation is:

$$\dot{X} = \left[N\left(\frac{P}{u(X / N_o)} \right) I - X \right] k \tag{2.21}$$

The actual timing of adoption may be delayed by factors such as searching for the "right store", arranging for financing, selling existing stock, were considered by Kalish. Therefore, the likelihood that a potential customer will adopt in the time interval dt is kdt, where k is the rate parameter (which could be a function of advertising, distribution, product quality and the like). If k is large, then timing of adoption immediately follows becoming a potential adopter, and vice versa.

Chapter 3

Development of Diffusion Model

3.1 Problem Formulation

The situation addressed by the present research is described as follows. A global firm is targeting the market of an emerging economy whose market has not yet been tapped for a particular consumer durable marketed by the firm. The consumer durable to be introduced by the firm in the local market has its presence in the market of developed economies for some time, but it is a new product and hence an innovation for the unexplored market. The local market of the emerging economy may consist of a fraction of population who are aware of the product or have already adopted the product through secondary channels such as acquaintances residing in open markets abroad.

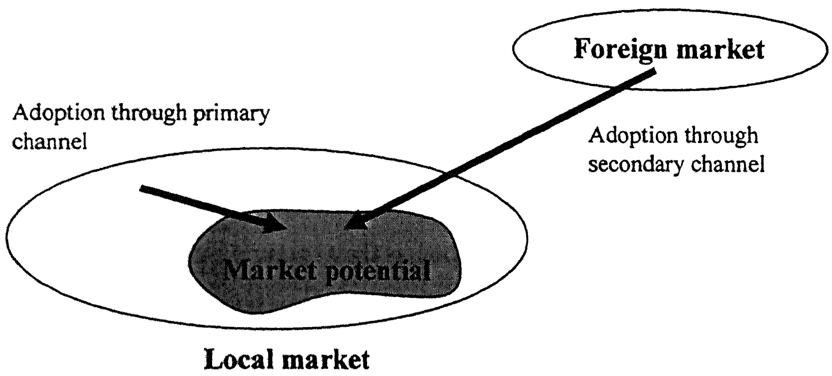


FIGURE 3.1. Primary and Secondary Channels of Adoption.

There is a portion of the total population in the local market depicted by shaded area in Figure 3.1, which comprises individuals who, once they become aware of the product features and price, will eventually adopt the product. This population of potential adopters constitutes the market potential for the product. Since primary channel for distribution does not exist, the consumers are compelled to rely on secondary channels for adopting the product. Before the restrictions on these markets are lifted and the product is made available, some adoptions will continue to take place through secondary channels. Even after the market opens and the product is available in the local market, adoption may still take place through secondary channels depending upon the price and quality differences of the product available through these two channels.

The major research question is - which advertising policy should the firm follow in order to maximize the total profits from adoption of the product from both the channels within a fixed planning period? It is to be noted here that different market conditions are prevalent during the two periods of the whole time horizon. Before opening of the market, the product is not available in the local market and only channel available for product adoption is through secondary channels by purchasing from foreign market. After opening of the local market, both primary and secondary channels exist. In addition to the above problem description, the current research seeks to study the effect of various parameters of the product diffusion model on the optimal advertising strategies as well as on awareness diffusion and product adoption over time.

3.2 Model Development

In the following model formulation, first purchase of a new consumer durable in a monopolistic market is considered. In the first-purchase diffusion models, one assumes that, in the product planning horizon being considered, there are no repeat purchases and purchase volume per buyer is one unit. Thus the product is bought once, if at all, by a member of the target population. Since a monopolist market is considered, there are no competitive responses to the actions taken by the firm.

Information flow is separated from the adoption process, a concept that has been applied before (Kalish 1985 and others). Once an individual has full information about the product, he will decide whether to adopt it or not. Further, the limiting case is considered in which consumers make purchase decision as soon as they become informed. It is assumed that the search-and-decision process is very fast in comparison with the time it takes for the diffusion process to develop. If the consumer decides not to adopt at that instant of time, in other words, postpones his decision of adoption, he will still remain a potential adopter and might take the adoption decision again at some other point in time. In what follows, the mathematical model for the product diffusion process is presented.

3.2.1 Awareness Diffusion

Information about the product will spread through a population of potential adopters. This flow of information is modeled as a diffusion process. The members of the potential buying population are assumed homogeneous with respect to the likelihood of being informed about the product and also for their propensity towards seeking information.

Let N_o be the relevant population size. This population is characterized by individuals who at some point in time will become aware about the existence and attributes of the product in the market either through advertising or word-of-mouth (WOM). The firm has to advertise about the innovation before its launch in the market so as to inform the consumers about the features and availability of the product. Advertising will have the effect of informing and motivating the innovators to adopt the product.

3.2.1.1 Incorporating Forgetting Effect

Mahajan and Muller (1986) have given the following expression to link advertising effectiveness to awareness diffusion:

$$\frac{dI}{dt} = f(A)[1 - I] - \delta I \qquad (3.1)$$

where,

I = fraction of the market aware of the product at any point in time

δ = decay or forgetting parameter.

The term $f(A)[1-I]$ in the above expression indicates "*Learning Effect*" and δI indicates "*Forgetting Effect*". An individual who has full information about the product may forget about the product attributes over time, therefore learning "over again" is necessary. However, the above equation does not include the effect of word of mouth.

The awareness diffusion equation to be used in the current research is modeled as follows:

$$\dot{I} = (1 - I)[f(A) + bI] - \delta I \qquad (3.2)$$

Here $f(A)$ indicates the effectiveness of the advertising expenditure rate (A) and the parameter 'b' takes into account the word of mouth effect. $\dot{I}$ is the rate of change of fraction of the population who are aware of the product by the time t. The term bI signifies the coefficient of internal influence due to word of mouth effect and is responsible for spread of awareness through communication.

This equation is similar to that proposed by Kalish (1985) with some modifications and simplification. Information diffusion is assumed to be homogeneous and an unaware individual can become aware only by word of mouth or by advertising. However to keep the later analysis tractable, actual adopters have not been considered separately in transmitting

information. It is also assumed that word of mouth communication is equally transmitted by the aware individuals irrespective of whether they have adopted the product or not.

3.2.1.2 Incorporating Advertising Effectiveness

The following behavior of advertising effectiveness function *f(A)* is expected as discussed by Kalish (1985):

$$f'(A) > 0, f(0) = 0, f''(A) < 0 \text{ and } A \geq 0.$$

The functional form of advertising effectiveness which satisfies the above requirement is assumed as follows:

$$f(A) = \beta \ln(1 + A) \tag{3.3}$$

$$\Rightarrow f'(A) = \frac{\beta}{1 + A} \tag{3.4}$$

$$\Rightarrow f''(A) = -\frac{\beta}{(1 + A)^2} \tag{3.5}$$

where β is a parameter and A is the advertising expenditure rate. A plot of the advertising effectiveness with respect to advertising expenditure rate is illustrated in Figure 3.2.

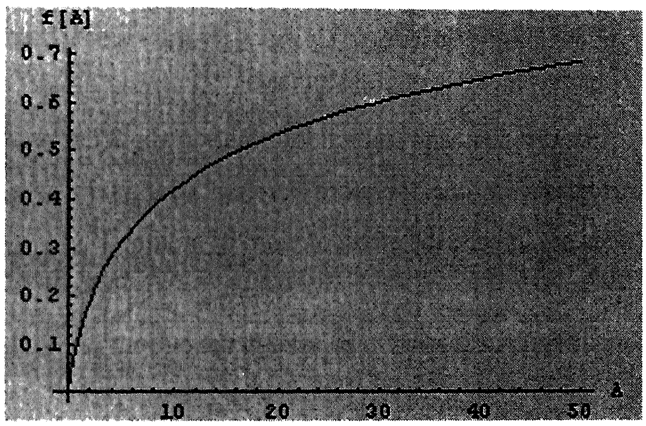


FIGURE 3.2. Advertising Effectiveness [*f(A)*].

3.2.2 Product Adoption

After becoming aware of the availability and features of the product, an individual will make the adoption decision, that is, whether to adopt the product or not. If the perceived value of the product is greater than or equal to the price, then the individual qualifies to be a potential adopter who might adopt the product eventually. The adoption decision process for a potential adopter can be considered as a Bernoulli trial which may result in either of two

outcomes; he adopts the product at that instant of time or does not, in other words, delays the adoption by choosing not to adopt at that time.

The sample space for a Bernoulli trial consists of two sample points, success and failure. In the present case, the event adoption of the product may be labeled as success and its complement event of non-adoption as failure. The actual adoption of the product by the whole population can be considered as independent Bernoulli trials by each consumer who at that moment is making the adoption decision. This is analogous to a Binomial experiment with 'n' trials and parameter 'p' denoting probability of success.

Thus the number of adoptions taking place in a small interval of time is given by the expected value of the Binomial variable.

Hence,

$$\dot{X} = np \tag{3.6}$$

where $\dot{X}$ is the adoption rate.

Here 'p' is the probability that the consumer makes the decision to purchase at that time and will depend on various factors such as price, quality, and, 'n' is the number of consumers who at that time interval are evaluating the product. Therefore, n is proportional to the total potential adopter population who are aware of the product but have not yet adopted it.

Therefore,

$$n \propto [NI - X]$$

where 'N' is the market potential.

So Equation (3.6) is given as follows:

$$\dot{X} = [NI - X]k \tag{3.7}$$

This adoption equation is similar to that proposed by Kalish (1985) however; additional effects need to be incorporated in Equation (3.7) as discussed later.

3.3 The Diffusion Model

Let t_o be the time (*refer Figure 3.3*) when the market opens and the product is made available in the local market by the firm. The market opening time (t_o) is taken to be deterministic in the present study and it is assumed that both the firm as well as the consumers are aware of it.

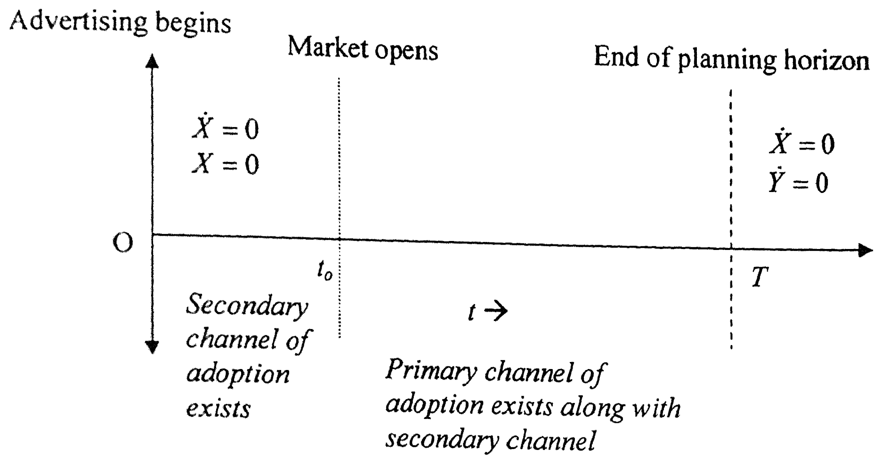


FIGURE 3.3. Channels of Adoption before and after Opening of Market.

Let T denote the end of planning period after which the firm will withdraw the product from the local market as well as from the foreign market due to technological obsolescence or to make way for a newer product. So the product will become unavailable for purchase after time T both through primary and secondary channel. Hence,

$$\dot{X}=0 \text{ and } \dot{Y}=0, \text{ for } t>T. \tag{3.8}$$

The planning period $(0 \le t < T)$ is divided into two periods by the market opening time (t_o) and has to be considered separately as different market conditions exist in each period. In what follows, awareness diffusion and product adoption equations are given for each period of time.

3.3.1 Before Market Opening $(0 \le t < t_o)$:

The awareness diffusion $I(t)$ is given by the following differential equation:

$$\dot{I}=(1-I)[f(A)+bI]-\delta I, \qquad I(0)=I_0. \tag{3.9}$$

Since primary channel does not exist, $\dot{X}=0$. Any adoption of the product is only through secondary channel from the foreign market. Since the product has been available in the foreign market for some time, the uncertainty associated with the performance of the product is negligible and need not be considered.

3.3.1.1 Incorporating Market Opening Time

The potential adopters may delay their decision to adopt the product as time approaches the market opening time (t_o), because of the anticipation that product available in the local market after the market opens, will be of less or equal price compared to that available through foreign channel and the quality might be same or better. This effect of time is captured by $\theta(t)$ in the adoption equation given below. Thus $\theta(t)$ should decrease as time approaches the market opening time (t_o). Further, the rate of change of $\theta(t)$ should decrease with time as illustrated in Figure 3.4.

Therefore,

$$\dot{\theta}(t) \leq 0, \ddot{\theta}(t) \leq 0.$$

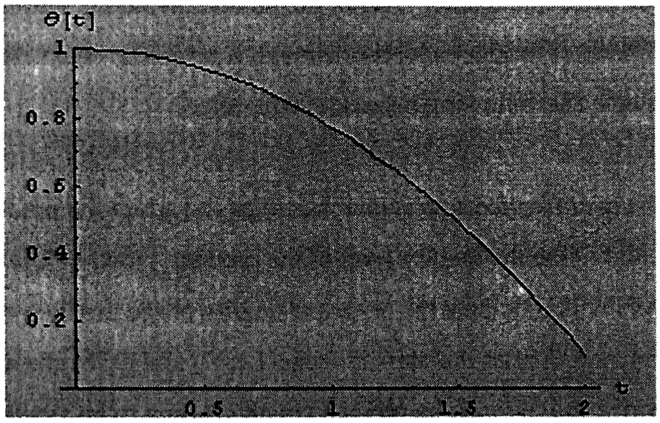


FIGURE 3.4. Plot of *θ(t)* with respect to *t*.

The following functional form for $\theta(t)$ is assumed which satisfies the above requirement.

$$\theta(t) = 1 - \varepsilon\left(\frac{t}{t_o}\right)^2 \tag{3.10}$$

$$\Rightarrow \dot{\theta}(t) = -\left(\frac{2\varepsilon}{t_o^2}\right)t \tag{3.11}$$

$$\Rightarrow \ddot{\theta}(t) = -\left(\frac{2\varepsilon}{t_o^2}\right) \tag{3.12}$$

where ε is a parameter.

---

[1] Subscript 'Y' stands for association of the quantity with the secondary channel of adoption.

The adoption through secondary channel is given as follows[1]:

$$\dot{Y} = [N_Y I - Y]\theta(t)k_1, \quad Y(0) = Y_0. \tag{3.13}$$

Here k_1 is the rate parameter such that $k_1 dt$ gives the likelihood that a potential customer will adopt in time dt, which depends on several factors such as price, quality and time. Thus k_1 is a function of anticipated price and quality of the product which is going to be available through primary channel with respect to the corresponding values for secondary channel.

3.3.1.2 Incorporating Price and Quality

A consumer can be categorized in the following two groups:

1. One, who has access to secondary channel, and thus will have an additional choice of product adoption besides through the primary channel,
2. Others, for whom the only source to adopt the product is through primary channel.

The difference in the price and quality level of the product available through the two channels will affect the likelihood of product adoption through a particular channel by the first category of consumers. The following behavior is expected.

Let P_Y be the overall price a consumer has to pay to adopt the product through secondary channel and Q_Y be the quality level of the product. Now, before opening of the market the consumers will anticipate the price and quality of the product which is going to be available in the local market after the market opens. Let P_a and Q_a be the anticipated values of price and quality respectively for the product which is going to be available through primary channel[2]. The following relation should hold regarding the behavior of likelihood of adoption (k_1):

$$\frac{dk_1}{d(P_Y/P_a)} < 0$$

$$\frac{dk_1}{d(Q_Y/Q_a)} > 0$$

---

[2] Subscript 'a' stands for anticipated value of the quantity.

As the price ratio (P_Y/P_a) decreases, consumers will anticipate a cheaper product to be available from the local market, so they may delay there adoption through secondary channel and the likelihood of adoption (k_1) should decrease. Similarly with decrease in quality ratio (Q_Y/Q_a), the likelihood of adoption (k_1) should increase, since the customers will anticipate a better quality product going to be available through the primary channel. The following functional form for k_1 is assumed to incorporate the effect of price and quality difference:

$$k_1 = e^{-\psi_1(.)} \tag{3.14}$$

where $\psi_1(.) = \alpha + \mu\frac{P_Y}{P_a} + v\frac{Q_a}{Q_Y}$ and α, μ, v are some parameters.

3.3.2 After Market Opening $(t_o < t \leq T)$:

After the market opens, product adoption will start taking place through primary channel. The awareness diffusion equation is considered the same as given by Equation (3.9) but the initial condition will now be different. Thus, $I(t_o)$ will be equal to the awareness level just before opening of the market and is obtained by solving Equation (3.9). Therefore,

$$\dot{I} = (1-I)[f(A)+bI] - \delta I, \quad I(t_0) = I_{t_0}. \tag{3.15}$$

Due to availability of the product in the local market, the market potential of the product will expand, as the product is now within the reach of greater number of consumers who do not have access to secondary channels. Some consumers might still use secondary channel for product adoption because of the uncertainty in the product quality available through primary channel.

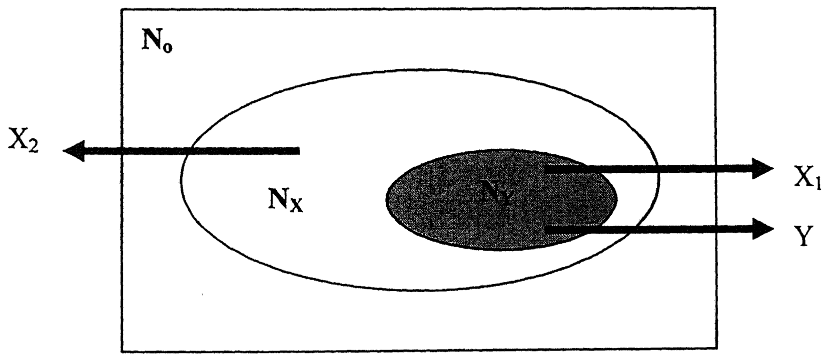


FIGURE 3.5. Market potential and Product Adoption after Market Opening.

In Figure 3.5,

N_o = total relevant potential,

N_Y = market potential of the product adopted through secondary channel,

N_X = market potential of the product adopted through primary channel[3],

Y = number of adopters in population N_Y who have adopted through secondary channel by time t,

X_1 = number of adopters in population N_Y who have adopted through primary channel by time t,

X_2 = number of adopters in population N_X - N_Y who have adopted through primary channel by time t.

The product adoption through the secondary channel is given as follows:

$$\dot{Y} = [N_Y I - (X_1 + Y)]k_2, \qquad Y(t_0) = Y_{t_0}. \tag{3.16}$$

Here (X_1+Y) takes into account the fact that after the market opens, product adoption by the population N_Y will be through both primary and secondary channel and hence the remaining potential adopter population which is aware of the product is $[N_Y I - (X_1+Y)]$.

3.3.2.1 Incorporating Uncertainty

Uncertainty about the product performance may be due to lack of information about the effectiveness or quality of the product. Since on average people can be considered as risk-averse, the uncertainty will often result in a postponement of the decision until further evidence can be gathered (Rogers 1995). The effect of uncertainty of product performance is incorporated in the diffusion model as suggested by Oren and Schwartz (1988) (*refer 2.3.2.2*).

The adoption rate is proportional to the number of potential adopters who have not adopted the product but have full information about it.

---

[3] Subscript 'X' stands for association of the quantity with the primary channel of adoption.

Thus, the adoption equation is given as,

$$\dot{X} = [NI - X]\Phi(X)k \tag{3.17}$$

where,

$$\Phi(X) = 1 - e^{-\omega(X+X_0+1)}. \tag{3.18}$$

Here ω is a parameter and X_0 is prior adoptions at $t = 0$.

$\Phi(X)$ gives the fraction of the total population with risk-aversion less than the risk-aversion threshold in terms of the cumulative adoptions. A larger threshold implies greater market share for the product. As the cumulative adoptions increase over time, threshold value increases and hence uncertainty declines (*refer 2.3.2.2*). This is depicted in Figure3.6. Therefore, $\Phi(X)$ should satisfy the following requirement:

$$\frac{d\Phi(X)}{dX} \geq 0$$

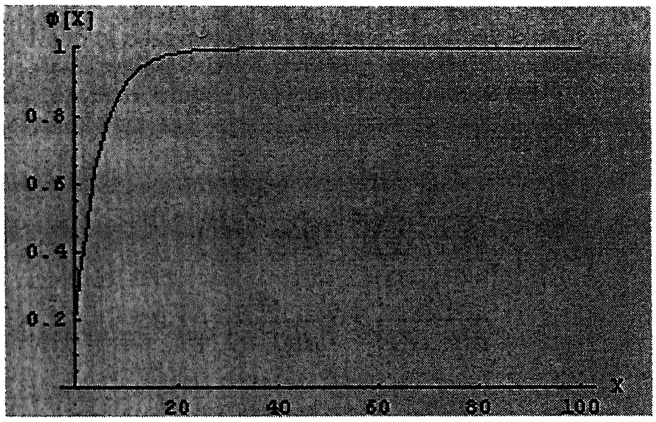


FIGURE 3.6. Plot of $\Phi(X)$ as a function of cumulative adoptions (X).

Similarly, the product adoption through the primary channel by the population N_Y is given as follows:

$$\dot{X}_1 = [N_Y I - (X_1 + Y)]\Phi(X_1 + X_2)k_3, \qquad X_1(t_0) = 0. \tag{3.19}$$

Here, $\Phi(X_1 + X_2)$ account for the uncertainty regarding product performance of the product available in local market, where (X_1+X_2) is the total adoption from the primary channel. The uncertainty will decrease after some time when a considerable amount of adoptions have taken place through primary channel. It is assumed that the product available through the two

different channels is essentially the same with similar features, though the firm may set a different price and quality level according to its policies.
The parameters k_2 and k_3 have the same interpretation as that for k_1 but will be a function of relative price and quality ratio of the product available through both the channels.

Finally, the consumers who do not have access to secondary channel is equal to $(N_X - N_Y)$ and the adoption by them is given as follows:

$$\dot{X}_2 = [(N_X - N_Y)I - X_2]\Phi(X_1 + X_2)k_4, \qquad X_2(t_0) = 0 \tag{3.20}$$

Here, $\Phi(X_1 + X_2)$ is the same function to incorporate uncertainty as used in product adoption Equation (3.19).

3.3.2.2 Incorporating Price and Quality

The effect of price and quality difference on the likelihood of adoption before opening of market will also be there after opening of the market for the first category of consumers as discussed in previous section. The consumers will now be aware of the actual price (P_X) and quality (Q_X) of the product and will decide whether to adopt through secondary channel or primary channel. As the price ratio (P_Y / P_X) increases, the likelihood of adoption through secondary channel (k_2) should decrease. Similarly with increase in quality ratio (Q_Y / Q_X), the likelihood of adoption through secondary channel (k_1) should increase. The following relations for k_1 and k_2 is expected:

$$\frac{dk_2}{d(P_Y/P_X)} < 0, \frac{dk_2}{d(Q_Y/Q_X)} > 0$$

$$\frac{dk_3}{d(P_Y/P_X)} > 0, \frac{dk_3}{d(Q_Y/Q_X)} < 0$$

The functional form for k_2 will be similar to k_1 as discussed in previous section but with different parameter values. Accordingly, k_3 is to be modified so that the inequalities hold. The functional form of k_4 is expected to be dependent on P_X and Q_X only as the consumers who do not have access to secondary channel will not be affected by the corresponding values for the product available in foreign market. The following relations for k_1 are expected:

$$\frac{dk_4}{dP_X} < 0, \frac{dk_4}{dQ_X} > 0.$$

That is, the likelihood of adoption (k_1) will decrease for increase in price and decrease in quality for the product available through primary channel.

Chapter 4

Diffusion Model Properties: Numerical Analysis

In this chapter, properties of the proposed diffusion model developed in the previous chapter are studied by assuming numerical values for the model parameters[4]. Numerical analysis is done using technical computing software *Mathematica 5*, to solve the model equations and generate plot of the variables as a function of time.

4.1 Effect of 'Forgetting' parameter (δ)

Substituting the following parameter values-

$$b = 0.327, f(A) = 0.164, k = 0.5746, N = 100, X_0 = 0, I_0 = 0.01$$

in the diffusion model:

$$\dot{I} = (1-I)[f(A) + bI] - \delta I, \qquad I(0) = I_0$$

$$\dot{X} = [NI - X]k, \qquad X(0) = X_0.$$

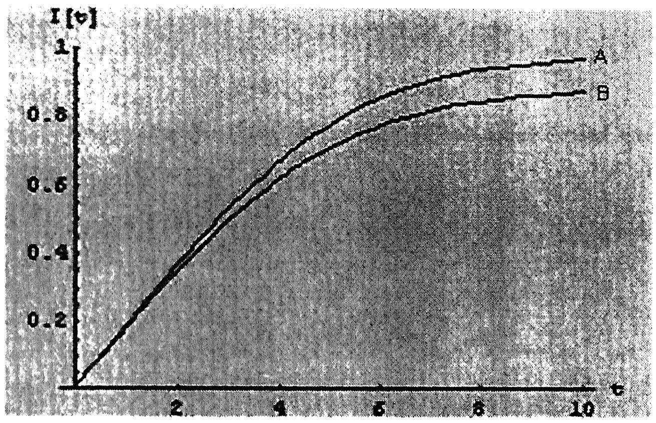


FIGURE 4.1(a). Awareness Diffusion with and without considering 'Forgetting' Effect.

In the Figure 4.1(a), curve **B** shows the effect of 'forgetting' parameter ($\delta = 0.05$) on the awareness diffusion whereas for curve **A**, forgetting effect was not considered ($\delta = 0$).

---

[4] Some values of the parameters are taken from the literature whereas others are assumed for analysis purpose. The numerical value for coefficient of internal influence (b) is taken from the estimates on penetration data collected in the United States for consumer electronics. The numerical value for likelihood of adoption (k) is taken from the estimated value used by Kalish (1985).

As expected curve **B** lies below curve **A**, showing decrease in awareness level with time due to forgetting by the consumers.

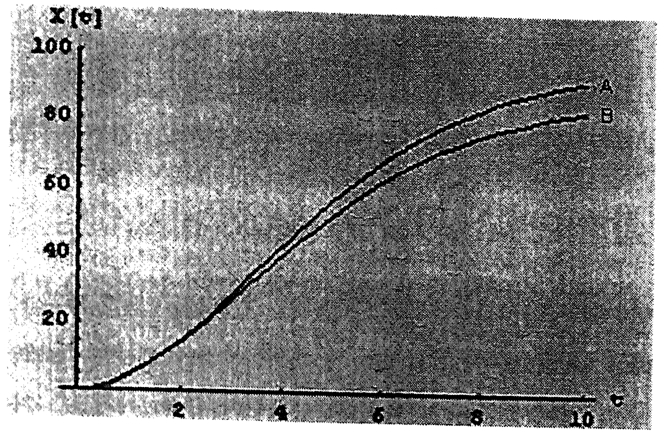


FIGURE 4.1(b). **Cumulative Adoptions (X) with and without considering 'Forgetting' Effect.**

Similarly in Figure 4.1(b), curve **B** shows lower cumulative adoptions due to lower awareness level in case of forgetting by the consumers as compared to curve **A**.

4.2 Effect of Coefficient of Internal Influence (b) and External Influence ($f(A)$)

Substituting the following parameter values-

$$b = 0.327,\ k = 0.5746,\ N = 100,\ X_0 = 0,\ I_0 = 0.01$$

in the diffusion model:

$$\dot{I} = (1 - I)[f(A) + bI] - \delta I, \qquad I(0) = I_0$$

$$\dot{X} = [NI - X]k, \qquad X(0) = X_0.$$

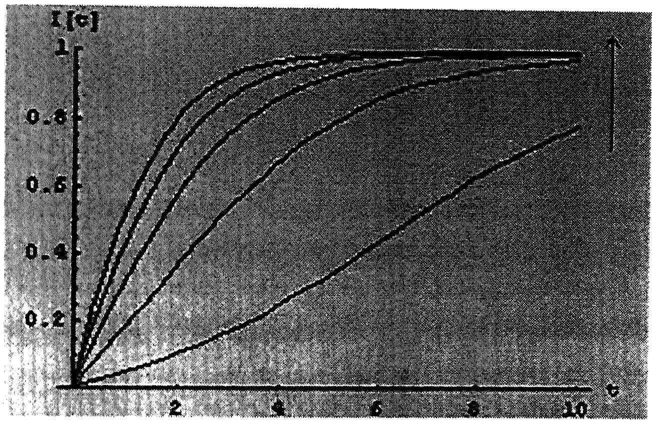


FIGURE 4.2(a). **Awareness Diffusion for various values of $f(A)$ = {0.01 b, 0.05 b, 0.1 b, 0.5 b, 1.0 b}.**

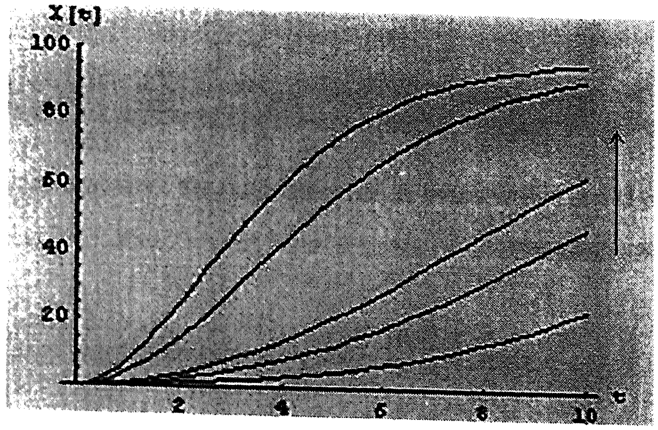


FIGURE 4.2(b). Cumulative Adoptions (X) for various values of $f(A)$ = {0.01 b, 0.05 b, 0.1 b, 0.5 b, 1.0 b}.

In Figure 4.2(a) and 4.2(b), the direction of the arrow indicates increasing value of $f(A)$ with which the curve is plotted. As expected a higher value of $f(A)$ results in a higher awareness diffusion rate and hence higher cumulative adoptions over time.

4.3 Effect of Market Opening Time (t_o)

Substituting the following parameter values-

$$b = 0.327, f(A) = 0.164, k_1 = 0.5746, \varepsilon = 0.9, N_Y = 15, t_o = 2, Y_0 = 0, I_0 = 0.01$$

in the diffusion model:

$$\dot{I} = (1 - I)[f(A) + bI], \qquad I(0) = I_0$$

$$\dot{Y} = [N_Y I - Y]\left[1 - \varepsilon\left(\frac{t}{t_o}\right)^2\right]k_1, \; Y(0) = Y_0.$$

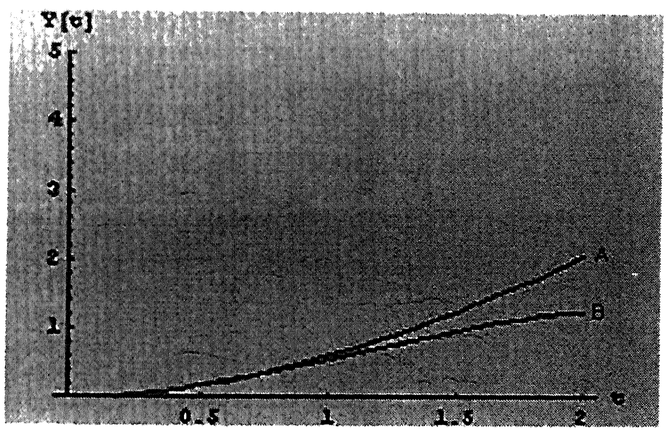


FIGURE 4.3(a). Cumulative Adoptions (Y) with and without considering effect of t_o.

In Figure 4.3(a), curve **B** takes into account the effect of market opening time (t_o) whereas curve **A** does not consider this effect. Since consumers postpone their adoption through secondary channel as time approaches t_o, hence curve **B** increasingly diverts from curve **A** over time. Also curve **B** lies below curve **A** due to decrease in adoption rate by considering the effect of market opening time.

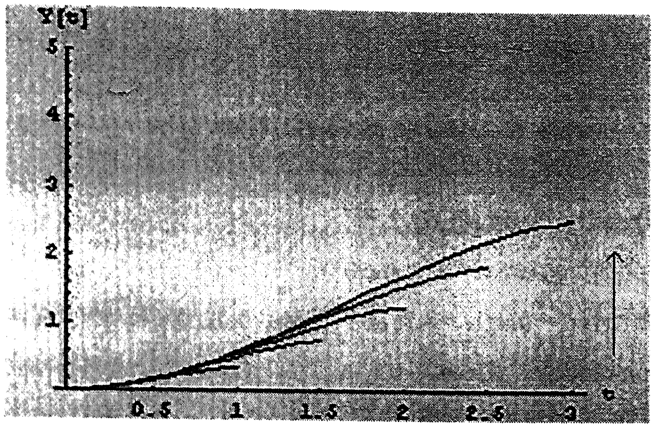


FIGURE 4.3(b): Cumulative Adoptions (Y) for various values of t_o = {1.0, 1.5, 2.0, 2.5, 3.0}.

Figure 4.3(b) shows the cumulative adoption curves for various values of t_o, where the direction of the arrow indicates increasing value of t_o. As expected, higher value of t_o imply less effect of t_o on product adoption rate, hence cumulative adoptions increase for higher values of t_o.

4.4 Effect of Price Ratio (P_Y/P_a)

Substituting the following parameter values-

$b = 0.327$, $f(A) = 0.164$, $\varepsilon = 0.9$, $\alpha = 0.7$, $\mu = 1.5$, $v = 0.02$, $Q_Y/Q_a = 1$, $N_Y = 15$, $Y_0 = 0$, $I_0 = 0.01$ in the diffusion model:

$$\dot{I} = (1 - I)[f(A) + bI], \qquad I(0) = I_0$$

$$\dot{Y} = [N_Y I - Y]\left[1 - \varepsilon\left(\frac{t}{t_o}\right)^2\right]e^{-\psi_1()}, \qquad Y(0) = Y_0.$$

where $\psi_1() = \alpha + \mu\dfrac{P_Y}{P_a} + v\dfrac{Q_a}{Q_Y}$.

The direction of the arrow in the Figure 4.4 indicates increasing value of (P_Y/P_a). Since a higher price for product available through secondary channel as compared to the anticipated

price for the same through primary channel will lower the product adoption rate through secondary channel, hence the direction of the arrow is downwards.

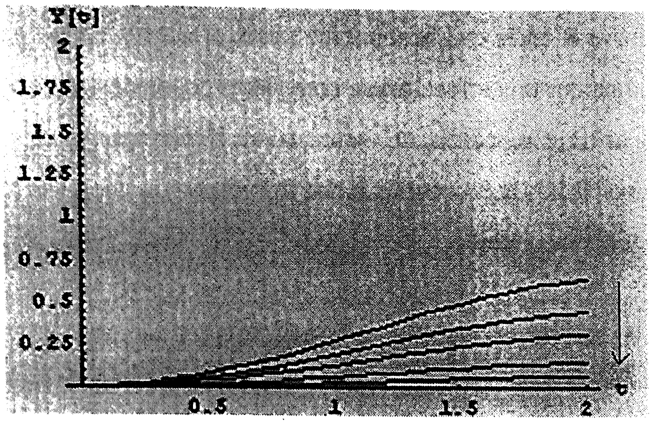


FIGURE 4.4. Cumulative Adoptions (Y) for various values of (P_Y/P_a) = {0.5, 0.75, 1.0, 1.5, 2.0}.

4.5 Effect of Quality Ratio (Q_Y/Q_a):

Substituting the following parameter values-

$b = 0.327$, $f(A) = 0.164$, $\varepsilon = 0.9$, $\alpha = 0.7$, $\mu = 1.5$, $v = 0.02$, $P_Y/P_a = 1$, $N_Y = 15$, $Y_0 = 0$, $I_0 =$ 0.01 in the diffusion model:

$$\dot{I} = (1 - I)[f(A) + bI], \qquad I(0) = I_0$$

$$\dot{Y} = [N_Y I - Y]\left[1 - \varepsilon\left(\frac{t}{t_o}\right)^2\right]e^{-\psi_1(.)}, \qquad Y(0) = Y_0.$$

where $\psi_1(.) = \alpha + \mu\frac{P_Y}{P_a} + v\frac{Q_a}{Q_Y}$.

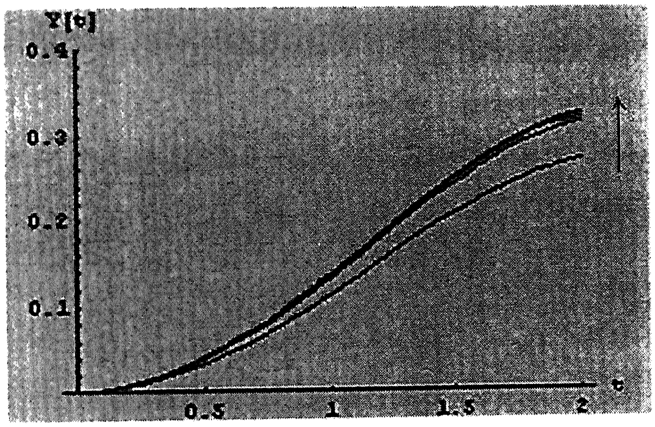


FIGURE 4.5. Cumulative Adoptions (Y) for various values of (Q_Y/Q_a) = {0.1, 0.5, 1.0, 3.0, 5.0}.

The direction of the arrow in the Figure 4.5 indicates increasing value of (Q_Y/Q_a). Since a higher quality level of product available through secondary channel as compared to the anticipated quality level for the same through primary channel will increase the product adoption rate through secondary channel, hence the direction of the arrow is upwards.

4.6 Effect of Uncertainty ($\Phi(X)$):

Substituting the following parameter values-

$b = 0.327, f(A) = 0.164, k_1 = 0.5746, \varepsilon = 0.9, N_Y = 15, t_o = 2, Y_0 = 0, I_0 = 0.01$

in the diffusion model:

For $0 \le t < t_o$,

$$\dot{I} = (1 - I)[f(A) + bI], \qquad I(0) = I_0.$$

$$\dot{Y} = [N_Y I - Y]\left[1 - \varepsilon\left(\frac{t}{t_o}\right)^2\right]k_1, \quad Y(0) = Y_0$$

and solving for $I(t_o)$ and $Y(t_o)$, the following values were obtained:

$Y(t_o) = 1.25495, I(t_o) = 0.368148.$

Then substituting the following parameter values-

$b = 0.327, f(A) = 0.164, N_Y = 15, N_X = 75, t_o = 2, k_2 = 0.2, k_3 = 0.4, k_4 = 0.5746$

in the diffusion model:

For $t_o < t \le T$,

$$\dot{I} = (1 - I)[f(A) + bI] - \delta I, \qquad I(t_0) = I_{t_0}$$

$$\dot{Y} = [N_Y I - (X_1 + Y)]k_2, \qquad Y(t_0) = Y_{t_0}$$

$$\dot{X}_1 = [N_Y I - (X_1 + Y)]\Phi(X_1 + X_2)k_3, \qquad X_1(t_0) = 0$$

$$\dot{X}_2 = [(N_X - N_Y)I - X_2]\Phi(X_1 + X_2)k_4, \qquad X_2(t_0) = 0$$

Taking,

$\Phi(X) = 1 - e^{-0.2(X+1)}$.

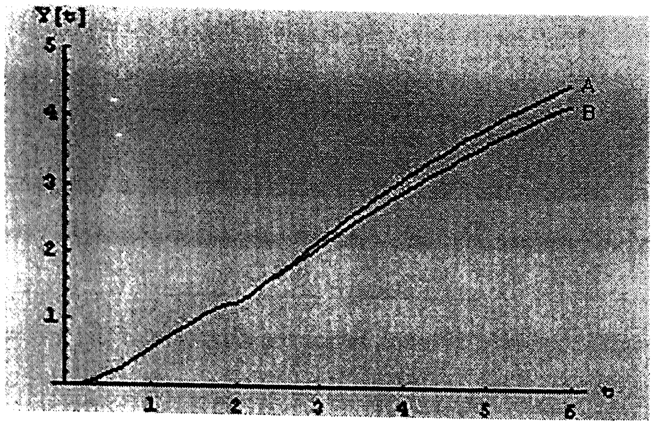


FIGURE 4.6(a). Cumulative Adoptions (Y) with and without considering Uncertainty.

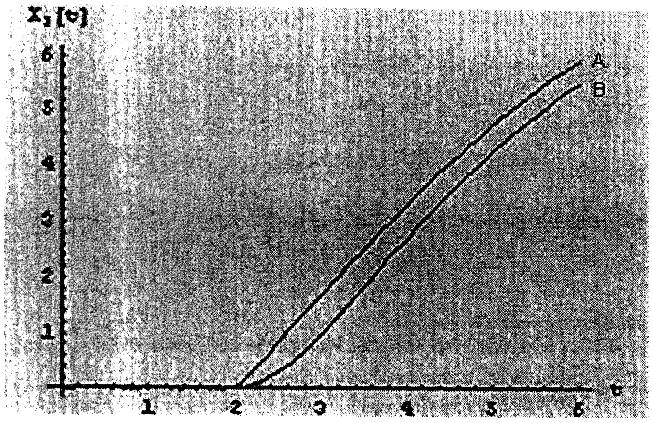


FIGURE 4.6(b). Cumulative Adoptions (X_1) with and without considering Uncertainty.

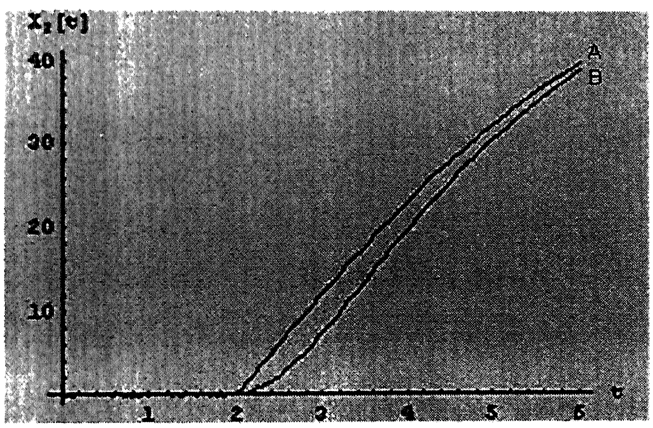


FIGURE 4.6(c). Cumulative Adoptions (X_2) with and without considering Uncertainty.

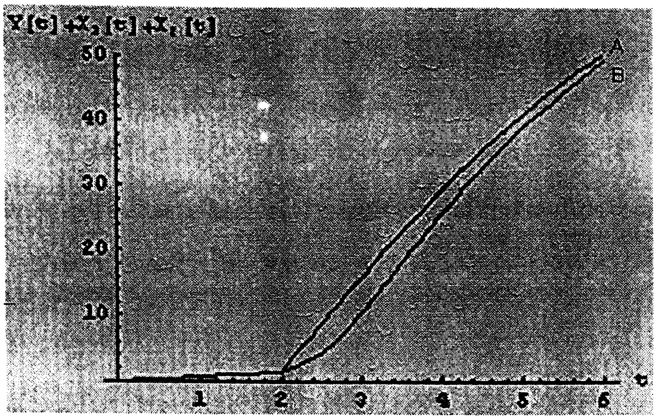


FIGURE 4.6(d). Total Cumulative Adoptions with and without considering Uncertainty.

In Figure 4.6(a), (b), (c) and (d), curve **B** shows cumulative adoptions over time considering uncertainty regarding product performance whereas curve **A** does not consider uncertainty. It is evident that uncertainty has the effect of delaying product adoption hence lower cumulative adoptions over time. It may also be noted that separation between curve **A** and **B** is more significant for cumulative adoptions through primary channel as compared to that for secondary channel. This is due to the fact that that effect of uncertainty is more prominent in case of $X_1(t)$ and $X_2(t)$ as compared to $Y(t)$ since uncertainty indirectly affects $Y(t)$ through $X_1(t)$.

4.7 Numerical Example

We now present a numerical example to illustrate the overall behavior of the diffusion model developed so far. All the functional forms assumed will be applied together in the diffusion model, which would help in understanding the awareness diffusion and product adoption for a practical situation.

The following parameters have been chosen for the diffusion model given below:

$b = 0.327, A = 60, \beta = 0.04, \delta = 0.05, I_0 = 0.01, Y_0 = 0, k_1 = 0.5, \varepsilon = 0.9, t_o = 2,$
$N_Y = 15, N_X = 75, \omega = 0.2, k_2 = 0.2, k_3 = 0.4, k_4 = 0.5746, T = 6.$

The Diffusion Model

For $0 \le t < t_o$,

$$\dot{I} = (1 - I)[\beta \ln(1 + A) + bI] - \delta I, \qquad I(0) = I_0.$$

$$\dot{Y} = [N_Y I - Y]\left[1 - \varepsilon\left(\frac{t}{t_o}\right)^2\right]k_1, \qquad Y(0) = Y_0.$$

For $t_o < t \le T$,

$$\dot{I} = (1 - I)[\beta \ln(1 + A) + bI] - \delta I, \qquad I(t_0) = I_{t_0}$$

$$\dot{Y} = [N_Y I - (X_1 + Y)]k_2, \qquad Y(t_0) = Y_{t_0}$$

$$\dot{X}_1 = [N_Y I - (X_1 + Y)](1 - e^{-\omega(X_1 + X_2 + 1)})k_3, \qquad X_1(t_0) = 0$$

$$\dot{X}_2 = [(N_X - N_Y)I - X_2](1 - e^{-\omega(X_1 + X_2 + 1)})k_4, \qquad X_2(t_0) = 0.$$

Technical computing software, *Mathematica 5* was used to solve the above simultaneous first order ordinary differential equations and to generate the following plots.

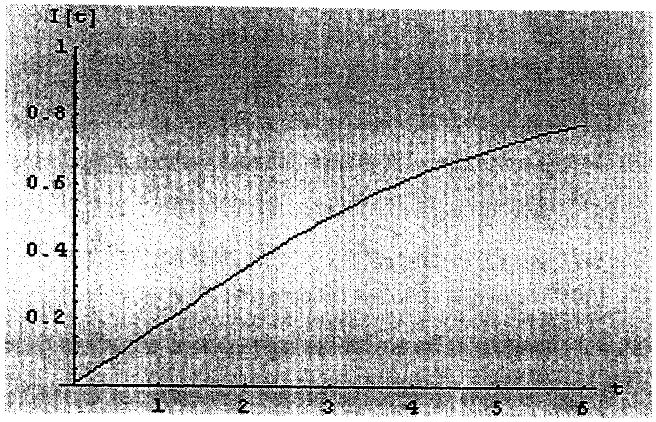


FIGURE 4.7(a). Awareness Diffusion for Whole Planning Period.

Figure 4.7(a) shows the awareness diffusion considering a constant level of advertising expenditure rate (A = 60) throughout the planning period. For the value of coefficient of internal influence (b = 0.327) chosen and advertising expenditure rate, almost 80% of the total population is aware of the product by the end of the planning horizon.

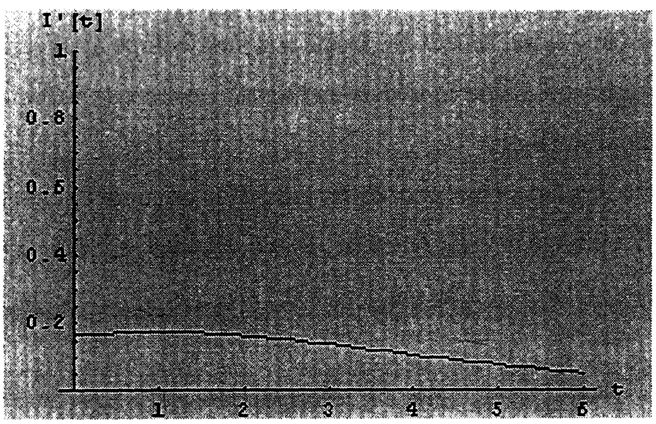


FIGURE 4.7(b). Awareness Diffusion Rate for Whole Planning Period.

The rate of awareness diffusion declines with time as evident from Figure 4.7(b). This is due to the fact that as awareness level increases (*see Figure 4.7(a)*), the remaining fraction of the population which is unaware about the product, decreases with time.

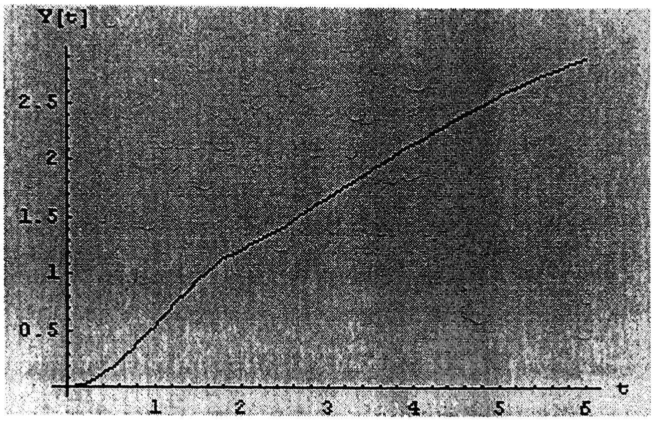


FIGURE 4.7(c). Cumulative Adoptions through Secondary Channel for Whole Planning Period.

It may be observed in Figure 4.7(c), that the slope of cumulative adoption curve for product adoption through secondary channel decreases after market opening ($t_o = 2$) as compared to that before t_o. This reflects the fact that some consumers who have access to secondary channels might adopt through primary channel, hence a decrease in adoption rate through secondary channel after market opening.

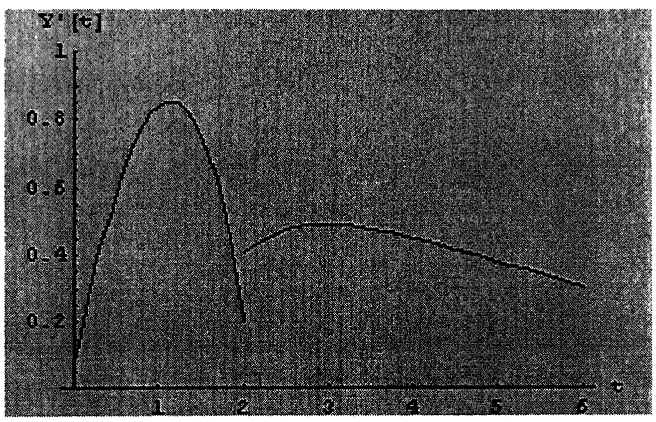


FIGURE 4.7(d). Adoption Rate through Secondary Channel for Whole Planning Period.

The adoption rate through secondary channel first increases due to increase in awareness level with time but later decreases as shown in Figure 4.7(d), as time approaches the market opening time ($t_o = 2$). This is due to the effect of consumer anticipation discussed in section 3.3.1.1. There is a break in the adoption rate curve at $t = t_o$, because of change in market

conditions around the market opening time. After the market opens, the adoption rate first increases little because of increase in awareness level and then declines steadily as the remaining potential adopter population who have not yet adopted, decreases with time.

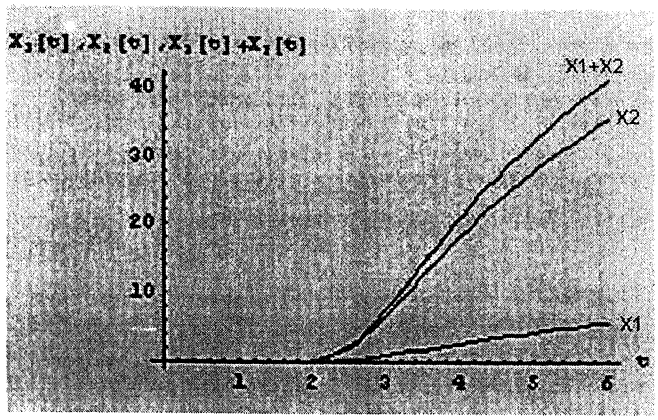


FIGURE 4.7(e). Cumulative Adoptions through Primary Channel for Whole Planning Period.

Figure 4.7(e) shows the plots **X1**, **X2** and **X1+X2** for cumulative adoptions $X_1(t)$, $X_2(t)$ and $X_1(t)+X_2(t)$ respectively. Since adoptions through primary channel begin only after market opens at $t_o = 2$, therefore the curves lie completely on the right side of t_o.

The adoption curves follow the standard S-shape of product diffusion. The curve **X1** lies very low in comparison to **X2** since the potential adopter's population who have access to secondary channel ($N_Y = 15$)is taken to be very less as compared to market potential of the local product ($N_X = 75$) in the numerical example.

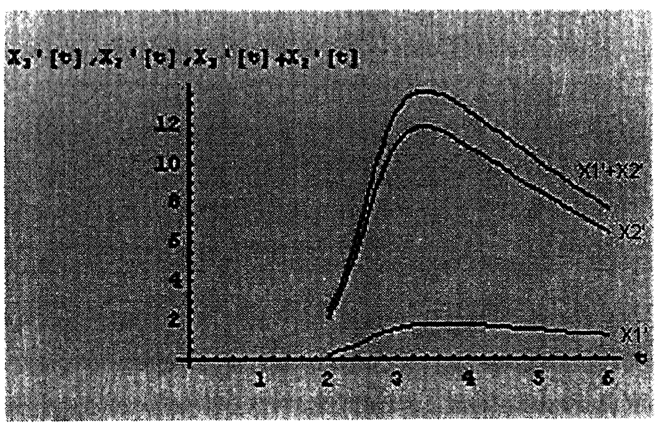


FIGURE 4.7(f). Adoption Rates through Primary Channel for whole Planning Period.

Similarly, Figure 4.7(f) shows the corresponding plots for adoption rates. The adoptions rates first increases due to increase in awareness level then gradually decrease, when the potential adopter population decreases significantly.

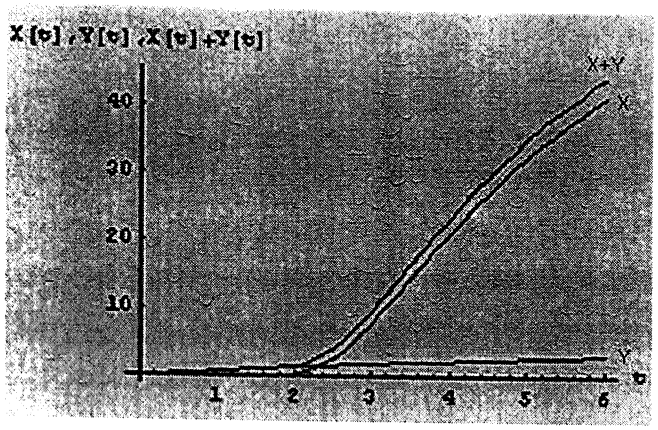


FIGURE 4.7(g). **Cumulative Adoptions through Primary and Secondary Channel.**

Figure 4.7(g) illustrates the cumulative adoptions through primary channel (*X*) and secondary channel (*Y*) as well as the total product adoption combined through both the channels (*X+Y*). It may be observed that for the numerical example setting chosen, the cumulative adoptions at the end of the planning period through each channel is far below the market potential (N_Y = 15, N_X = 75).

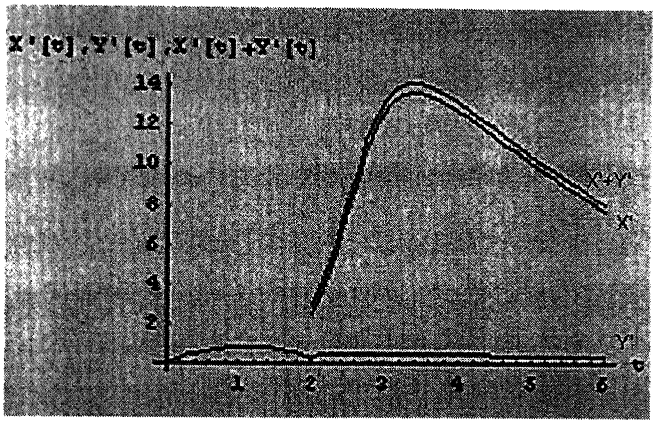


FIGURE 4.7(h). **Adoption Rates through Primary Channel for whole Planning Period.**

Figure 4.7(h) illustrates the adoption rates through primary channel and secondary channel and also the total product adoption rate. It may be observed that adoption rate through secondary channel becomes negligible at the end of the planning period because of a very low likelihood of adoption after opening of the market. The total product adoption rate is

same as that of the adoption rate through secondary channel before market opening. But there is a break at ($t = t_o$) due to the changes in the market conditions around market opening time ($t_o = 2$).

Chapter 5

Optimal Control Approach for Advertising Policy: Propositions

5.1 Optimal Control Model

A firm can partially control the sales of its product by varying the level of advertising. Based on the results obtained on studying the diffusion model properties, it is evident that advertising helps in diffusion of awareness, which in turn affects the rate of product adoption positively. The firm will have to decide the allocation of resources for advertising which will lead to maximization of discounted profits over the planning horizon. Since a higher level of advertising will result in higher sales rate, but will also incur higher advertising expenditure rate, hence an optimal level of advertising expenditure rate has to be determined.

We propose to determine the optimal advertising policy which a decision-maker should follow when faced with a situation described in Chapter 3. The diffusion model proposed in Chapter 4 is used to apply optimal control technique developed by Pontryagin (1962). The Pontryagin's maximum principle for optimal control is a generalization of calculus of variations (*see Kamien and Schwartz (1981)*). In optimal control problem, variables are divided into two classes: state variables and control variables. The movement of the state variable is governed by first order differential equations. The control variable influences the objective function both directly and indirectly (through its impact on the evolution of the state variable). We formulate the optimal control problem as described below.

The optimization problem is to identify the optimal advertising policy *A(t)* which will maximize:

$$\pi = \int_0^{t_o} \left\{ e^{-rt} \left[g_Y \dot{Y} - A \right] \right\} dt + \int_{t_o}^{T} \left\{ e^{-rt} \left[g_X \left(\dot{X}_1 + \dot{X}_2 \right) + g_Y \dot{Y} - A \right] \right\} dt \qquad (5.1.1)$$

where π is the net discounted profits over the planning horizon and

r = discount rate,

g_Y= gross profit for unit product sale through secondary channel,

g_X= gross profit for unit product sale through primary channel,

T = length of the planning period.

Here the state variables are $Y(t)$, $X_1(t)$, and $X_2(t)$, and the control variable is $A(t)$. The argument t will frequently be suppressed. All the variables and parameters have the same interpretation as defined earlier in the diffusion model formulation. It is assumed that the gross margin exclusive of advertising expenditure, g_X and g_Y, are constants which, do not vary in time. Thus, the difference between the two components of the gross margin, price and variable costs, remains unchanged throughout the product life cycle.

The *state equations* (sometimes called *transition equations*) along with their endpoint conditions for the optimal control problem are given as follows:

For $0 \le t < t_o$,

$$\dot{I} = (1-I)[f(A)+bI]-\delta I, \qquad I(0)=I_0 \tag{5.1.2}$$

$$\dot{Y} = [N_Y I - Y]\theta(t)k_1, \qquad Y(0)=Y_0. \tag{5.1.3}$$

For $t_o < t \le T$,

$$\dot{I} = (1-I)[f(A)+bI]-\delta I, \qquad I(t_0)=I_{t_0} \tag{5.1.4}$$

$$\dot{Y} = [N_Y I - (X_1+Y)]k_2, \qquad Y(t_0)=Y_{t_0} \tag{5.1.5}$$

$$\dot{X}_1 = [N_Y I - (X_1+Y)]k_3, \qquad X_1(t_0)=0 \tag{5.1.6}$$

$$\dot{X}_2 = [(N_X - N_Y)I - X_2]k_4, \quad X_2(t_0)=0 \tag{5.1.7}$$

Further, the upper limit on the advertising expenditure rate is set by managerial consideration; therefore the control in the problem is bounded.

$$0 \le A \le \bar{A} \text{ for } 0 \le t \le T \tag{5.1.8}$$

Necessary Conditions for Optimality –

In the optimal control technique, the above problem is equivalent to maximizing the *current value Hamiltonians* for the two time periods, which are defined below:

For $0 \le t < t_o$,

$$\begin{aligned} H_1 &= \{g_Y \dot{Y} - A\} + m_{1Y}\dot{Y} + m_{1I}\dot{I} \\ &= -A + (g_Y + m_{1Y})\dot{Y} + m_{1I}\dot{I} \end{aligned} \tag{5.1.9}$$

For $t_o < t \leq T$,

$$H_2 = \{g_X(\dot{X}_1 + \dot{X}_2) + g_Y\dot{Y} - A\} + m_{2X_1}\dot{X}_1 + m_{2X_2}\dot{X}_2 + m_{2Y}\dot{Y} + m_{2I}\dot{I}$$
$$= -A + (g_X + m_{2X_1})\dot{X}_1 + (g_X + m_{2X_2})\dot{X}_2 + (g_Y + m_{2Y})\dot{Y} + m_{2I}\dot{I} \qquad (5.1.10)$$

There is a continuously differentiable *current value multiplier* function associated with each state equation such that the maximizing solution $A^*(t)$, $Y^*(t)$, $X_1^*(t)$, and $X_2^*(t)$ along with the multiplier functions simultaneously satisfy the state equations.

The *multiplier* (*costate, auxiliary, adjoint*) *equations* are as follows –

For $0 \leq t < t_o$,

$$\dot{m}_{2I} = rm_{2I} - \frac{\partial H_2}{\partial I} \qquad (5.1.11)$$

$$\dot{m}_{1Y} = rm_{1Y} - \frac{\partial H_1}{\partial Y} \qquad (5.1.12)$$

For $t_o < t \leq T$,

$$\dot{m}_{2I} = rm_{2I} - \frac{\partial H_2}{\partial I} \qquad (5.1.13)$$

$$\dot{m}_{2Y} = rm_{2Y} - \frac{\partial H_2}{\partial Y} \qquad (5.1.14)$$

$$\dot{m}_{2X_1} = rm_{2X_1} - \frac{\partial H_2}{\partial X_1} \qquad (5.1.15)$$

$$\dot{m}_{2X_2} = rm_{2X_2} - \frac{\partial H_2}{\partial X_2} \qquad (5.1.16)$$

The *optimality condition* –

For a maximization problem it is necessary that $A^*(t)$ maximize the *current value Hamiltonians* with respect to A. Therefore,

For $0 \leq t < t_o$,

$$\frac{\partial H_1}{\partial A} = 0 \qquad (5.1.17)$$

For $t_o < t \le T$,

$$\frac{\partial H_2}{\partial A} = 0 \qquad (5.1.18)$$

The following *transversality conditions* have to be satisfied by the *current value multipliers* -

$$m_{2I}(T) = 0 \qquad (5.1.19)$$

$$m_{2Y}(T) = 0 \qquad (5.1.20)$$

$$m_{2X_1}(T) = 0 \qquad (5.1.21)$$

$$m_{2X_2}(T) = 0 \qquad (5.1.22)$$

In addition to the standard conditions for optimality, the multipliers and the Hamiltonians are continuous at t_o, therefore

$$H_1(t_o) = H_2(t_o) \qquad (5.1.23)$$

$$m_{1I}(t_o) = m_{2I}(t_o) \qquad (5.1.24)$$

$$m_{1Y}(t_o) = m_{2Y}(t_o) \qquad (5.1.25)$$

5.2 Optimal Advertising Policy

Optimal advertising rate $A(t)$ at any time t is chosen so that for $j = 1,2$;

$$A(t) = 0 \quad \text{if } \frac{\partial H_j}{\partial A} < 0 \text{ at } t,$$

$$A(t) = \bar{A} \quad \text{if } \frac{\partial H_j}{\partial A} > 0 \text{ at } t,$$

$$0 < A(t) < \bar{A} \quad \text{if } \frac{\partial H_j}{\partial A} = 0 \text{ at } t.$$

Where $\dfrac{\partial H_j}{\partial A} = -1 + m_{jI}\dfrac{\partial \dot{I}}{\partial A}$.

If $\dfrac{\partial H_j}{\partial A} = 0$ then $A(t)$ is obtained by solving the following simultaneous Ordinary Differential Equations:

For $0 \le t < t_o$ (*refer Appendix A.1*),

$$\dot{I} = (1-I)[f(A) + bI] - \delta I, \qquad I(0) = I_0$$

$$-\frac{f''(A)}{f'(A)}\dot{A} = \left[r + \frac{\delta}{1-I}\right] - (1-I)[b + (g_Y + m_{1Y})N_Y \theta(t) k_1 f'(A)].$$

(5.2.1)

Where,

$$\left[m_{1Y} e^{-rt - k_1 \int \theta(t) dt}\right]_t^{t_o} = g_Y k_1 \int_t^{t_o} \left[\theta(t) e^{-rt - k_1 \int \theta(t) dt}\right] dt$$

Assuming $\theta(t) = 1$,

$$m_{1Y} = m_{1Y}(t_o) e^{-(r+k_1)(t_0 - t)} + \frac{g_Y k_1}{r + k_1}\left[e^{-(r+k_1)(t_0 - t)} - 1\right] \tag{5.2.2}$$

Where,

$$m_{1Y}(t_o) = \frac{g_Y k_2 + g_X k_3}{r + k_2 + k_3}\left[e^{-(r+k_2+k_3)(T - t_o)} - 1\right] \tag{5.2.3}$$

For $t_o < t \le T$ (*refer Appendix A.2*),

$$\dot{I} = (1-I)[f(A) + bI] - \delta I, \qquad I(t_0) = I_{t_0}$$

$$-\frac{f''(A)}{f'(A)}\dot{A} = \left[r + \frac{\delta}{1-I}\right] -$$

$$(1-I)\left[b + \{(g_X + m_{2X_1})N_Y k_3 + (g_X + m_{2X_2})(N_X - N_Y)k_4 + (g_Y + m_{2Y})N_Y k_2\} f'(A)\right]$$

(5.2.4)

Where,

$$m_{2X_1} = \frac{g_Y k_2 + g_X k_3}{r + k_2 + k_3}\left[e^{-(r+k_2+k_3)(T-t)} - 1\right] \tag{5.2.5}$$

$$m_{2X_2} = \frac{g_X k_4}{r + k_4}\left[e^{-(r+k_4)(T-t)} - 1\right] \tag{5.2.6}$$

$$m_{2Y} = \frac{g_Y k_2 + g_X k_3}{r + k_2 + k_3}\left[e^{-(r+k_2+k_3)(T-t)} - 1\right] \tag{5.2.7}$$

Although an analytical closed form solution for A^* as a function of time and the different parameters is difficult to obtain due to the complexity of the equations, the characteristics of the optimal advertising policy can be determined under mild conditions.

5.2.1 Before Market Opening $(0 \le t < t_o)$:

THEOREM 1.

Define

$$\chi_1 = \left[r + \frac{\delta}{1-I}\right] - (1-I)\left[b + (g_Y + m_{1Y})N_Y\theta(t)k_1 f'(A)\right] \tag{5.2.1.1}$$

Where χ_1 is a measure of the rate of change of advertising expenditure rate before opening of the market (*refer Equation (5.2.1)*).

Then it is optimal to,

i) increase advertising expenditure rate A(t) at time t, if

$$\chi_1 > 0$$

ii) decrease advertising expenditure rate A(t) at time t, if

$$\chi_1 < 0$$

iii) keep steady advertising expenditure rate A(t) at time t, if

$$\chi_1 = 0.$$

PROOF.

We know that *A(t)* is,

i) increasing if $\dot{A} > 0$,

ii) decreasing if $\dot{A} < 0$, and

iii) constant if $\dot{A} = 0$.

For $0 \le t < t_o$ (*refer Equation (5.2.1)*),

$$-\frac{f''(A)}{f'(A)}\dot{A} = \chi_1 \tag{5.2.1.2}$$

Since $f'(A) > 0, f''(A) < 0$ therefore sign of χ_1 is same as that of $\dot{A}$.

Hence, $A(t)$ is,

i) increasing if $\chi_1 > 0$,

ii) decreasing if $\chi_1 < 0$, and

iii) constant if $\chi_1 = 0$.

PROPOSITION 1. In an undiscounted case ($r = 0$) for a market situation before opening of the market where there are no significant effects of forgetting and market opening time (i.e., $\delta = 0$, $\theta(t) = 1$), and gross profit margin per unit through secondary channel (g_Y) is greater than the same through primary channel (g_X), that is $(g_Y > g_X)$, then optimal advertising is monotonic decreasing over time.

PROOF.

From Equation (5.2.2),

$$m_{1Y} = m_{1Y}(t_o)e^{-(r+k_1)(t_0-t)} + \frac{g_Y k_1}{r+k_1}\left[e^{-(r+k_1)(t_0-t)} - 1\right]$$

$$\Rightarrow g_Y + m_{1Y} = \left[m_{1Y}(t_o) + \frac{g_Y k_1}{r+k_1}\right]e^{-(r+k_1)(t_0-t)} + \frac{g_Y r}{r+k_1} \qquad (5.2.1.3)$$

Where (*refer Equation* (*5.2.3*)),

$$m_{1Y}(t_o) = \frac{g_Y k_2 + g_X k_3}{r+k_2+k_3}\left[e^{-(r+k_2+k_3)(T-t_o)} - 1\right]$$

Since $0 < e^{-(r+k_2+k_3)(T-t_o)} < 1$ therefore,

$$-\frac{g_Y k_2 + g_X k_3}{r+k_2+k_3} < m_{1Y}(t_o) < 0.$$

Thus,

$$\frac{g_Y k_1}{r+k_1} - \frac{g_Y k_2 + g_X k_3}{r+k_2+k_3} < m_{1Y}(t_o) + \frac{g_Y k_1}{r+k_1} < \frac{g_Y k_1}{r+k_1} \qquad (5.2.1.4)$$

Substituting the limiting values in Equation (5.2.1.3),

$$\left[\frac{g_Y k_1}{r+k_1} - \frac{g_Y k_2 + g_X k_3}{r+k_2+k_3}\right]e^{-(r+k_1)(t_0-t)} + \frac{g_Y r}{r+k_1} < g_Y + m_{1Y} < \left[\frac{g_Y k_1}{r+k_1}\right]e^{-(r+k_1)(t_0-t)} + \frac{g_Y r}{r+k_1}$$

Now if $\frac{g_Y k_1}{r+k_1} - \frac{g_Y k_2 + g_X k_3}{r+k_2+k_3} > 0$ (5.2.1.5)

$$\Rightarrow g_Y + m_{1Y} > 0$$

Putting $r = 0$ in Equation (5.2.1.5) then,

$$g_Y > \frac{g_Y k_2 + g_X k_3}{k_2 + k_3}$$

Simplifying,

$$g_Y > g_X$$

$$\Rightarrow g_Y + m_{1Y} > 0 \qquad (5.2.1.6)$$

Taking $r = 0$, $\delta = 0$, and $\theta(t) = 1$ in Equation (5.2.1.2),

Therefore,

$$\chi_1 = -(1-I)[b + (g_Y + m_{1Y})N_Y k_1 f'(A)]$$

Since $g_Y + m_{1Y} > 0$ from Equation (5.2.1.6), also $f''(A) > 0$ and all the parameters are greater than zero, therefore $\chi_1 < 0$. From Theorem 1, $\chi_1 < 0$ implies $\dot{A} < 0$.

The implication of Proposition 1 is that, if it is profitable for the firm to have its product adopted through secondary channel as compared to that from primary channel (since $g_Y > g_X$), then the optimal advertising policy is to do maximum advertising in the beginning of the planning period so that maximum adoptions takes place through secondary channel before opening of the market.

5.2.2 Around Market Opening $(t \approx t_o)$:

THEOREM 2.

Define

$$\chi_2 = \{g_X + m_{2X_1}(t_o)\}N_Y k_3 + \{g_X + m_{2X_2}(t_o)\}(N_X - N_Y)k_4 + \{g_Y + m_Y(t_o)\}N_Y[k_2 - \theta(t_o)k_1] \qquad (5.2.2.1)$$

and,

$\dot{A}(t_o^{+})$ = rate of change of advertising expenditure rate just after opening of the market,

$\dot{A}(t_o^-)$ = rate of change of advertising expenditure rate just before opening of the market. Where χ_2 is a measure of the difference in rate of change of advertising expenditure rate before and after opening of the market (*refer Equation* (5.2.1) *and* (5.2.4)).

Then it is optimal to have,

i) $\dot{A}(t_o^+) > \dot{A}(t_o^-)$, if

$$\chi_2 < 0$$

ii) $\dot{A}(t_o^+) < \dot{A}(t_o^-)$, if

$$\chi_2 > 0$$

iii) $\dot{A}(t_o^+) = \dot{A}(t_o^-)$, if

$$\chi_2 = 0.$$

PROOF.

From Equation (5.1.24)

$$m_{1I}(t_o) = m_{2I}(t_o)$$

$$\therefore \frac{1}{\left(1 - I_{t_o^-}\right) f'[A(t_o^-)]} = \frac{1}{\left(1 - I_{t_o^+}\right) f'[A(t_o^+)]}$$

But $I_{t_o^-} = I_{t_o^+} = I_{t_o}$

$$\therefore f'[A(t_o^-)] = f'[A(t_o^+)]$$

$$\Rightarrow A(t_o^-) = A(t_o^+) = A_{t_o} \qquad (5.2.2.2)$$

For $0 \le t < t_o$ (*refer Equation* (5.2.1)),

$$-\frac{f''(A)}{f'(A)}\dot{A} = \left[r + \frac{\delta}{1-I}\right] - (1-I)\left[b + (g_Y + m_{1Y})N_Y\theta(t)k_1 f'(A)\right]$$

Now at $t \to t_o^-, A = A(t_o^-)$ and $I = I_{t_o}$

$$\therefore -\frac{f''(A_{t_o})}{f'(A_{t_o})}\dot{A}(t_o^-) = \left[r + \frac{\delta}{1-I_{t_o}}\right] - \left(1-I_{t_o}\right)\left[b + (g_Y + m_{1Y}(t_o))N_Y\theta(t_o)k_1 f'(A_{t_o})\right]$$

(5.2.2.3)

For $t_o < t \leq T$ (*refer Equation* (5.2.4)),

$$-\frac{f''(A)}{f'(A)}\dot{A} = \left[r + \frac{\delta}{1-I}\right] -$$

$$(1-I)\left[b + \left\{\left(g_X + m_{2X_1}\right)N_Y k_3 + \left(g_X + m_{2X_2}\right)\left(N_X - N_Y\right)k_4 + \left(g_Y + m_{2Y}\right)N_Y k_2\right\}f'(A)\right]$$

(5.2.2.4)

Now at $t \to t_o^+, A = A(t_o^+)$ and $I = I_{t_o}$

$$\therefore -\frac{f''(A_{t_o})}{f'(A_{t_o})}\dot{A}(t_o^+) = \left[r + \frac{\delta}{1-I_{t_o}}\right] -$$

$$(1-I_{t_o})\left[b + \left\{\left(g_X + m_{2X_1}(t_o)\right)N_Y k_3 + \left(g_X + m_{2X_2}(t_o)\right)\left(N_X - N_Y\right)k_4 + \left(g_Y + m_{2Y}(t_o)\right)N_Y k_2\right\}f'(A_{t_o})\right]$$

(5.2.2.5)

From Equation (5.2.2.4) and Equation (5.2.2.5),

$$-\frac{f''(A_{t_o})}{f'(A_{t_o})}\left[\dot{A}(t_o^+) - \dot{A}(t_o^-)\right] =$$

$$-(1-I_{t_o})\left[\left\{\left(g_X + m_{2X_1}(t_o)\right)N_Y k_3 + \left(g_X + m_{2X_2}(t_o)\right)\left(N_X - N_Y\right)k_4 + \left(g_Y + m_Y(t_o)\right)N_Y\left(k_2 - \theta(t_o)k_1\right)\right\}f'(A_{t_o})\right]$$

Where $m_Y(t_o) = m_{1Y}(t_o) = m_{2Y}(t_o)$.

Or,

$$-\frac{f''(A_{t_o})}{f'(A_{t_o})}\left[\dot{A}(t_o^+) - \dot{A}(t_o^-)\right] = -(1-I_{t_o})\left[\chi_2 f'(A_{t_o})\right] \qquad (5.2.2.6)$$

Since $f'(A) > 0, f''(A) < 0$ and $\left(1 - I_{t_o}\right) \geq 0$, therefore sign of χ_2 is opposite to that of $\dot{A}(t_o^+) - \dot{A}(t_o^-)$.

Hence,

a) $\dot{A}(t_o^+) > \dot{A}(t_o^-)$, if

$$\chi_2 < 0$$

b) $\dot{A}(t_o^{+}) < \dot{A}(t_o^{-})$, if

$$\chi_2 > 0$$

c) $\dot{A}(t_o^{+}) = \dot{A}(t_o^{-})$, if

$$\chi_2 = 0.$$

PROPOSITION 2. For a market situation if the ratio of gross profit margin per unit through primary channel by that through secondary channel, satisfies the following inequality:

a) $$\frac{g_X}{g_Y} < \frac{(\theta(t_o)k_1 - k_2)}{(k_3 - k_4) + \frac{N_X}{N_Y}k_4} \tag{5.2.2.7}$$

then it is optimal to have the rate of change of advertising expenditure rate after market opening, more than that before market opening, that is, $\dot{A}(t_o^{+}) > \dot{A}(t_o^{-})$.

b) $$\frac{g_X}{g_Y} > \frac{\theta(t_o)k_1k_3 - r(k_2 - \theta(t_o)k_1)}{k_3[r + \theta(t_o)k_1]} \tag{5.2.2.8}$$

then it is optimal to have the rate of change of advertising expenditure rate after market opening, less than that before market opening, that is, $\dot{A}(t_o^{+}) < \dot{A}(t_o^{-})$.

PROOF.

Putting $t = t_o$ in Equations (5.2.5) to (5.2.7),

$$m_{2X_1}(t_o) = \frac{g_Y k_2 + g_X k_3}{r + k_2 + k_3}\left[e^{-(r+k_2+k_3)(T-t_o)} - 1\right]$$

$$m_{2X_2}(t_o) = \frac{g_X k_4}{r + k_4}\left[e^{-(r+k_4)(T-t_o)} - 1\right]$$

$$m_{2Y}(t_o) = \frac{g_Y k_2 + g_X k_3}{r + k_2 + k_3}\left[e^{-(r+k_2+k_3)(T-t_o)} - 1\right]$$

Since $0 < e^{-(r+k_2+k_3)(T-t_o)} < 1$ therefore,

$$-\frac{g_Y k_2 + g_X k_3}{r + k_2 + k_3} < m_{2X_1}(t_o) < 0.$$

Similarly,

$$-\frac{g_X k_4}{r+k_4} < m_{X_2}(t_o) < 0 \text{ , and}$$

$$-\frac{g_Y k_2 + g_X k_3}{r+k_2+k_3} < m_{2Y}(t_o) < 0.$$

Therefore (*refer Equation* (*5.2.2.1*)),

$$\min < \chi_2 < \max$$

Where,

$$\max = \{g_X N_Y k_3 + g_X (N_X - N_Y) k_4 + g_Y N_Y (k_2 - \theta(t_o) k_1)\} \tag{5.2.2.9}$$

and

$$\min = \left\{\frac{N_Y}{r+k_2+k_3}\left([g_X r + (g_X - g_Y)k_2]k_3 + [g_Y r + (g_Y - g_X)k_3](k_2 - \theta(t_o)k_1)\right) + \frac{g_X r}{r+k_4}(N_X - N_Y)k_4\right\} \tag{5.2.2.10}$$

(a)

On rearranging Equation (5.2.2.9),

$$\max = g_X [N_Y (k_3 - k_4) + N_X k_4] + g_Y N_Y (k_2 - \theta(t_o)k_1)$$

Now, if *max* < 0,

then,

$$\Rightarrow \frac{g_X}{g_Y} < \frac{(\theta(t_o)k_1 - k_2)}{(k_3 - k_4) + \frac{N_X}{N_Y} k_4}$$

Or,

$$\chi_2 < 0$$

Therefore from Theorem 2,

$$\dot{A}(t_o^+) > \dot{A}(t_o^-).$$

The implication of this result is that, if the ratio of gross profit margin per unit (g_Y / g_X) through the two channels is less than a critical value given by the inequality (5.2.2.7), then it is optimal to change advertising expenditure rate at a higher rate after opening of the market compared to that before market opening.

(b)

On rearranging Equation (5.2.2.10),

$$\min = [g_X r + (g_X - g_Y)k_2]k_3 + [g_Y r + (g_Y - g_X)k_3](k_2 - \theta(t_o)k_1)$$

$$\Rightarrow \min = [g_X(r + k_2) - g_Y k_2]k_3 + [g_Y(r + k_3) - g_X k_3](k_2 - \theta(t_o)k_1)$$

Or,

$$\min = g_X k_3[r + \theta(t_o)k_1] + g_Y[r(k_2 - \theta(t_o)k_1) - \theta(t_o)k_1 k_3]$$

Now, if $min > 0$,

$$\Rightarrow g_X k_3[r + \theta(t_o)k_1] > g_Y[\theta(t_o)k_1 k_3 - r(k_2 - \theta(t_o)k_1)]$$

then,

$$\frac{g_X}{g_Y} > \frac{\theta(t_o)k_1 k_3 - r(k_2 - \theta(t_o)k_1)}{k_3[r + \theta(t_o)k_1]}.$$

Or,

$$\chi_2 > 0$$

Therefore from Theorem 2,

$$\dot{A}(t_o^{+}) < \dot{A}(t_o^{-}).$$

The implication of this result is that, if the ratio of gross profit margin per unit (g_Y / g_X) through the two channels is greater than a critical value given by the inequality (5.2.2.8), then it is optimal to change advertising expenditure rate at a lower rate after opening of the market compared to that before market opening.

5.2.3 After Market Opening $(t_o < t \le T)$:

THEOREM 3.

Define

$$\chi_3 = \left[r + \frac{\delta}{1-I}\right] - (1-I)\left[b + \{(g_X + m_{2X_1})N_Y k_3 + (g_X + m_{2X_2})(N_X - N_Y)k_4 + (g_Y + m_{2Y})N_Y k_2\}f'(A)\right] \tag{5.2.3.1}$$

Where χ_3 is a measure of the rate of change of advertising expenditure rate after opening of the market (*refer Equation (5.2.4)*).

Then it is optimal to,

i) increase advertising expenditure rate A(t) at time t, if

$$\chi_3 > 0$$

ii) decrease advertising expenditure rate A(t) at time t, if

$$\chi_3 < 0$$

iii) keep steady advertising expenditure rate A(t) at time t, if

$$\chi_3 = 0.$$

PROOF.

We know that *A(t)* is,

i) increasing if $\dot{A} > 0$,

ii) decreasing if $\dot{A} < 0$, and

iii) constant if $\dot{A} = 0$.

For $t_o < t \le T$ (*refer Equation (5.2.4)*),

$$-\frac{f''(A)}{f'(A)}\dot{A} = \chi_3 \qquad (5.2.3.2)$$

Since $f'(A) > 0, f''(A) < 0$ therefore sign of χ_3 is same as that of $\dot{A}$.

Hence, *A(t)* is,

i) increasing if $\chi_3 > 0$,

ii) decreasing if $\chi_3 < 0$, and

iii) constant if $\chi_3 = 0$.

PROPOSITION 3. In an undiscounted case (*r* = *0*) for a market situation after opening of the market where there is no significant effect of forgetting (*δ* = *0*), then optimal advertising is monotonic decreasing over time.

PROOF.

From Equations (5.2.5) to (5.2.7),

$$m_{2X_1} = \frac{g_Y k_2 + g_X k_3}{r + k_2 + k_3}\left[e^{-(r+k_2+k_3)(T-t)} - 1\right]$$

$$m_{X_2} = \frac{g_X k_4}{r + k_4}\left[e^{-(r+k_4)(T-t)} - 1\right]$$

$$m_{2Y} = \frac{g_Y k_2 + g_X k_3}{r + k_2 + k_3}\left[e^{-(r+k_2+k_3)(T-t)} - 1\right]$$

Since $0 < e^{-(r+k_2+k_3)(T-t)} < 1$ therefore,

$$-\frac{g_Y k_2 + g_X k_3}{r + k_2 + k_3} < m_{2X_1} < 0. \tag{5.2.3.3}$$

Similarly,

$$-\frac{g_X k_4}{r + k_4} < m_{X_2} < 0, \text{ and} \tag{5.2.3.4}$$

$$-\frac{g_Y k_2 + g_X k_3}{r + k_2 + k_3} < m_{2Y} < 0. \tag{5.2.3.5}$$

Therefore,

$$\min < \{(g_X + m_{2X_1})N_Y k_3 + (g_X + m_{2X_2})(N_X - N_Y)k_4 + (g_Y + m_{2Y})N_Y k_2\} < \max \tag{5.2.3.6}$$

Substituting limiting values in Equation (5.2.3.6) and simplifying,

$$\max = \{g_X N_Y k_3 + g_X (N_X - N_Y)k_4 + g_Y N_Y k_2\}, \text{ and}$$

$$\min = \left\{\frac{N_Y r}{r + k_2 + k_3}(g_X k_3 + g_Y k_2) + \frac{g_X r}{r + k_4}(N_X - N_Y)k_4\right\}$$

Since $\min > 0$ as all the terms in the expression are positive therefore,

$$(1-I)\left[b + \{(g_X + m_{2X_1})N_Y k_3 + (g_X + m_{2X_2})(N_X - N_Y)k_4 + (g_Y + m_{2Y})N_Y k_2\}f'(A)\right] > 0 \tag{5.2.3.7}$$

Taking $r = 0$ and $\delta = 0$ then,

$$\chi_3 = -(1-I)\left[b + \{(g_X + m_{2X_1})N_Y k_3 + (g_X + m_{2X_2})(N_X - N_Y)k_4 + (g_Y + m_{2Y})N_Y k_2\}f'(A)\right] \tag{5.2.3.8}$$

From Equation (5.2.3.7) and (5.2.3.8),

$$\chi_3 < 0.$$

From Theorem 3, $\chi_1 < 0$ implies $\dot{A} < 0$.

Chapter 6

Comparative Assessment of Various Advertising Policies: Numerical Analysis

In this section, various illustrative numerical examples are examined to demonstrate the results regarding optimal advertising policies obtained in the previous section. The theorems developed so far indicate that the optimal advertising policy, that is, whether to increase or decrease the advertising expenditure rate at a time, depends on the numerical values of the various parameters of the model. Thus, different market situations will require different advertising policies for maximization of profit at the end of the planning horizon.

Here we will first consider different market situations by assuming numerical values for the model parameters. We would then find out the optimal advertising policy for each situation. The methodology used is to iteratively generate the values of different variables to finally arrive at the net profit at the end of the planning horizon. The optimal advertising policy is found by enumerating all possible advertising policies for a small example setting and then comparing the net profit for each advertising policy for the particular market situation. The discrete version of the diffusion model equations proposed earlier is given below.

For $0 \le t < t_o$,

$$\frac{\Delta I_t}{\Delta t} = (1 - I_t)[\beta \ln(1 + A_t) + bI_t] - \delta I_t, \qquad I_0 = I(0).$$

$$\frac{\Delta Y_t}{\Delta t} = [N_Y I_t - Y_t]\left[1 - \varepsilon\left(\frac{t}{t_o}\right)^2\right]k_1, \qquad Y_0 = Y(0)$$

$$\frac{\Delta X_{1t}}{\Delta t} = 0, \qquad X_{10} = 0$$

$$\frac{\Delta X_{2t}}{\Delta t} = 0, \qquad X_{20} = 0$$

For $t_o < t \le T$,

$$\frac{\Delta I_t}{\Delta t} = (1 - I_t)[\beta \ln(1 + A_t) + bI_t] - \delta I_t, \qquad I_{t_0} = I(t_0)$$

$$\frac{\Delta Y_t}{\Delta t} = [N_Y I_t - (X_{2t} + Y_t)]k_2, \qquad Y_{t_0} = Y(t_0)$$

$$\frac{\Delta X_{1t}}{\Delta t} = [N_Y I_t - (X_{1t} + Y_t)]k_3, \qquad X_{1t_o} = 0$$

$$\frac{\Delta X_{2t}}{\Delta t} = [(N_X - N_Y)I_t - X_{2t}]k_4, \qquad X_{2t_o} = 0.$$

The following equations are used to update the value of the variables in each period.

$$I_{t+1} = I_t + \Delta I_t$$

$$Y_{t+1} = Y_t + \Delta Y_t$$

$$X_{1t+1} = X_{1t} + \Delta X_{1t}$$

$$X_{2t+1} = X_{2t} + \Delta X_{2t}$$

The net profit at any time t is given by π_t, where

$$\pi_t = \pi_{t-1} + \left(e^{-rt}\left[g_Y \Delta Y_t + g_X(\Delta X_1 + \Delta X_2) - A_t \Delta t\right]\right), \quad \pi_0 = 0.$$

The entire planning horizon (T) is divided into number of small incremental time elements of equal size (Δt) given by: $\Delta t = T/n$

where n is the total number of divisions of the planning period. A higher value of n will result in a greater accuracy in the iterated calculations.

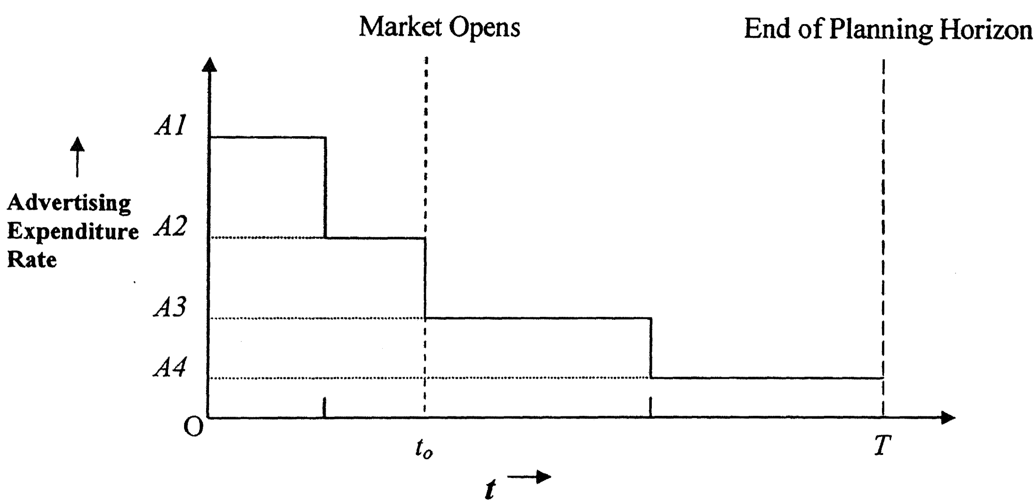


FIGURE 6.1. Operationalisation of Advertising Policy.

A policy is operationalised as (A1,A2,A3,A4) where A1, A2, A3 and A4 are the levels at which the advertising expenditure rate is maintained during that period. In the current numerical settings, the total planning period is divided into four parts, in which the period before market opening is divided into two parts of equal time duration and the period after market opening is also divided into two equal parts. It may be noted that, since the period before market opening may not be of the same length as that after market opening in the example setting, hence the length of the parts before and after market opening may be unequal. As depicted in Figure 6.1, A1 and A2 are the advertising expenditure level before market opening, and A3 and A4 are the advertising expenditure level after market opening for the two subsequent periods.

In the following numerical examples, values of the parameters have been assumed for numerical analysis[4].

6.1 Effect of Discount rate (r) and Forgetting rate (δ):

In this numerical example, we will compare the optimal advertising policies for the following two cases. First, specific numerical values are assumed for discount rate and forgetting rate in the diffusion model, whereas in the second case they are taken to be zero. All the other model parameters have the same values for both the cases.

(a) $(r \neq 0, \delta \neq 0)$:

We choose the following parameters-

N_X	N_Y	r	g_Y	g_X	β	b	δ	I(0)	ε	Y(0)	k_1	k_2	k_3	k_4	t_o	T
75	15	0.3	10	7	0.04	0.327	0.2	0.01	0.9	0	0.5	0.2	0.4	0.5746	2	6

Iterating the values of the variables for $n = 120$ time periods, therefore $\Delta t = T/120 = 0.05$ we obtain the following optimal advertising policy (*refer Table B.1.1*).

A1	A2	A3	A4		I	Y	X_1	X_2	$X=X_1+X_2$	X+Y	π
0	25	25	0		0.362	1.689	2.952	18.234	21.186	22.875	21.195

The optimal advertising policy (0, 25, 25, 0) is depicted in Figure 6.1(a).

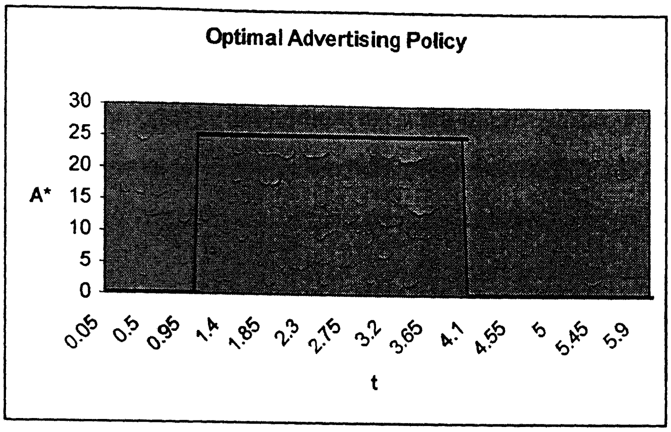


FIGURE 6.1(a). Plot of Optimal Advertising Policy.

(b) $(r = 0, \delta = 0)$:

We choose the following parameters-

N_X	N_Y	r	g_Y	g_X	β	b	δ	I(0)	ε	Y(0)	k_1	k_2	k_3	k_4	t_o	T
75	15	0	10	7	0.04	0.327	0	0.01	0.9	0	0.5	0.2	0.4	0.5746	2	6

Iterating the values of the variables for $n = 120$ time periods, therefore $\Delta t = T/120 = 0.05$ we obtain the following optimal advertising policy (*refer Table B.1.2*).

A1	A2	A3	A4		I	Y	X_1	X_2	$X=X_1+X_2$	X+Y	π
25	25	25	0		0.747	3.606	5.284	34.707	39.991	43.597	218.543

The optimal advertising policy (25, 25, 25, 0) is depicted in Figure 6.1(b).

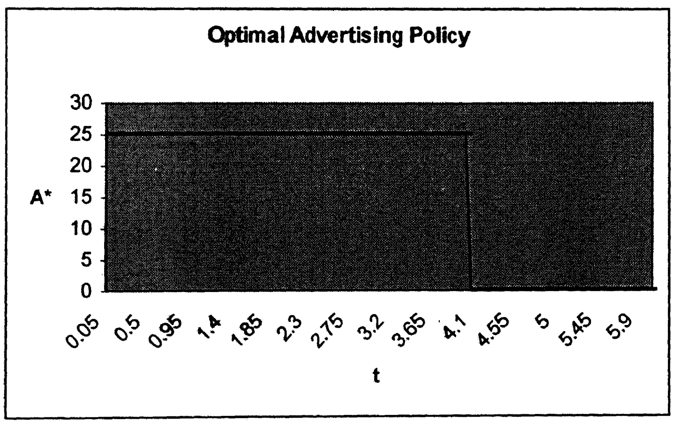


FIGURE 6.1(b). Plot of Optimal Advertising Policy.

If discounting is considered, future values of rewards and expenditures are discounted back to time 0. In the first case where $(r \neq 0, \delta \neq 0)$, it is optimal to gradually increase the advertising

expenditure before market opening. Since time value of money is considered, it is beneficial to spend on advertising at a later time as the value of money decreases with time. Whereas for the case in which $(r = 0, \delta = 0)$, the optimal advertising policy is to maintain it at some level and then gradually decrease after opening of the market. These contrasting policies for the two cases can be explained by Theorem 1 (*refer 6.2.1*). By taking significant value of discount rate and forgetting rate in the diffusion model, χ_1 becomes positive for $t < t_o$, hence according to Theorem 1, it is optimal to increase advertising expenditure rate.

6.2 Effect of Advertising Expenditure rate (A):

In this numerical example setting we will compare the optimal advertising policies for the following two cases. In the first case, the firm can afford a high advertising expenditure rate whereas in the second case, the upper limit on advertising expenditure rate ($\bar{A}$) is low. All the other model parameters have the same values for both the cases.

(a) $\bar{A}$ is set at a high level:

We choose the following parameters-

N_X	N_Y	r	g_Y	g_X	β	b	δ	I(0)	ε	Y(0)	k_1	k_2	k_3	k_4	t_o	T
75	15	0	10	7	0.04	0.327	0	0.01	0.9	0	0.5	0.2	0.4	0.5746	2	6

Iterating the values of the variables for $n = 120$ time periods, therefore $\Delta t = T / 120 = 0.05$ we obtain the following optimal advertising policy (*refer Table B.2.1*).

A1	A2	A3	A4		I	Y	X_1	X_2	$X=X_1+X_2$	X+Y	π
50	50	0	0		0.670	3.380	4.516	30.746	35.262	38.642	183.010

The optimal advertising policy (50, 50, 0, 0) is depicted in Figure 6.2(a).

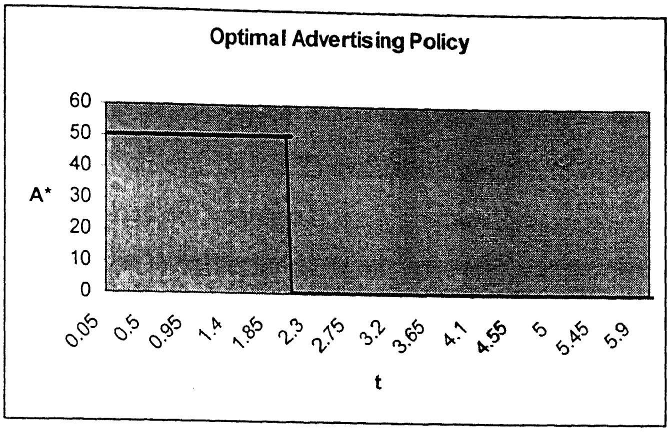


FIGURE 6.2(a). Plot of Optimal Advertising Policy.

(b) $\bar{A}$ is set at a low level:

We choose the following parameters-

N_X	N_Y	r	g_Y	g_X	β	b	δ	I(0)	ε	Y(0)	k_1	k_2	k_3	k_4	t_o	T
75	15	0	10	7	0.04	0.327	0	0.01	0.9	0	0.5	0.2	0.4	0.5746	2	6

Iterating the values of the variables for n = 120 time periods, therefore $\Delta t = T / 120 = 0.05$ we obtain the following optimal advertising policy (*refer Table B.2.2*).

A1	A2	A3	A4
10	10	10	0

I	Y	X_1	X_2	$X=X_1+X_2$	X+Y	π
0.667	3.061	4.617	29.982	34.599	37.659	235.334

The optimal advertising policy (10, 10, 10, 0) is depicted in Figure 6.2(b).

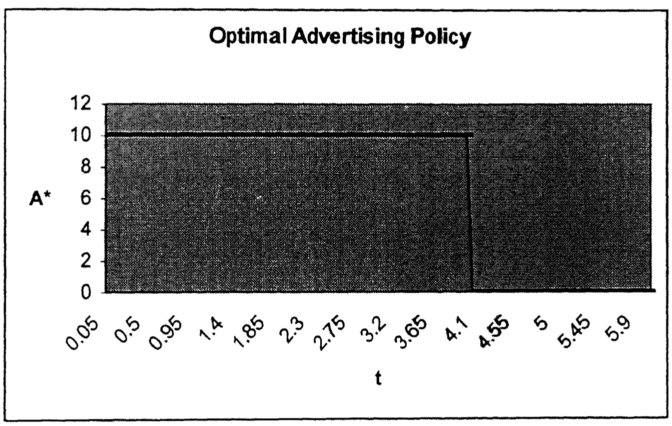


FIGURE 6.2(b). Plot of Optimal Advertising Policy.

We observe that the optimal advertising policy for both the cases above is to maintain the advertising expenditure rate at a steady level and then gradually decrease it. This can be

explained by Proposition 1 (*refer 6.2.1*) and Proposition 3 (*refer 6.2.3*). Since $g_Y > g_X$ and also $r = 0$ and $\delta = 0$, hence according to Proposition 1 and 2, it is optimal to decrease advertising expenditure rate gradually. But the difference in the length of advertising period for the two cases is due to the fact that for a low advertising rate, it takes longer for the awareness to reach a significant level.

6.3 Effect of Length of Planning Horizon (T):

In this numerical example setting we will compare the optimal advertising policies for the following two cases. In the first case, the planning period is set much larger in comparison to the market opening time. In the second case, the end of planning period is closer to the market opening time. Thus, the product will be available for adoption for a longer time in the first case as compared to that for the second case. All the other model parameters have the same values for both the cases.

(a) $T >> t_o$:

We choose the following parameters-

N_X	N_Y	r	g_Y	g_X	β	b	δ	I(0)	ε	Y(0)	k_1	k_2	k_3	k_4	t_o	T
75	15	0	10	7	0.04	0.327	0	0.01	0.9	0	0.5	0.2	0.4	0.5746	2	6

Iterating the values of the variables for $n = 120$ time periods, therefore $\Delta t = T / 120 = 0.05$ we obtain the following optimal advertising policy (*refer Table B.3.1*).

A1	A2	A3	A4
25	25	25	0

I	Y	X_1	X_2	$X=X_1+X_2$	X+Y	π
0.747	3.606	5.284	34.707	39.991	43.597	218.543

The optimal advertising policy (25, 25, 25, 0) is depicted in Figure 6.3(a).

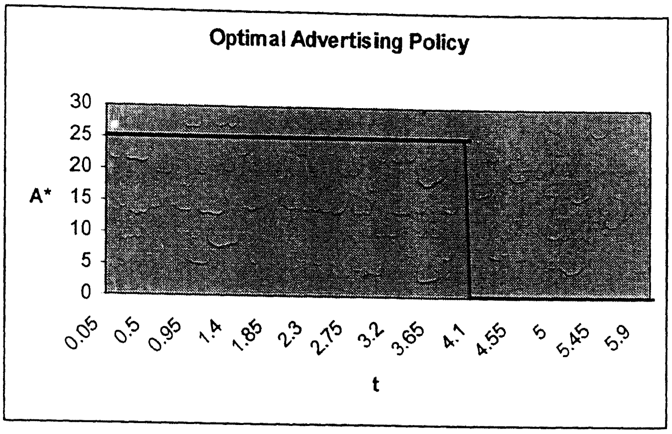


FIGURE 6.3(a). Plot of Optimal Advertising Policy.

(b) $T > t_o$:

We choose the following parameters-

N_X	N_Y	r	g_Y	g_X	β	b	δ	I(0)	ε	Y(0)	k_1	k_2	k_3	k_4	t_o	T
75	15	0	10	7	0.04	0.327	0	0.01	0.9	0	0.5	0.2	0.4	0.5746	2	4

Iterating the values of the variables for $n = 120$ time periods, therefore $\Delta t = T / 120 = 0.05$ we obtain the following optimal advertising policy (*refer Table B.3.2*).

A1	A2	A3
25	25	0

I	Y	X1	X2	X=X1+X2	X+Y	π
0.460	2.127	2.326	16.273	18.599	20.726	104.287

The optimal advertising policy (25, 25, 0) is depicted in Figure 6.3(b).

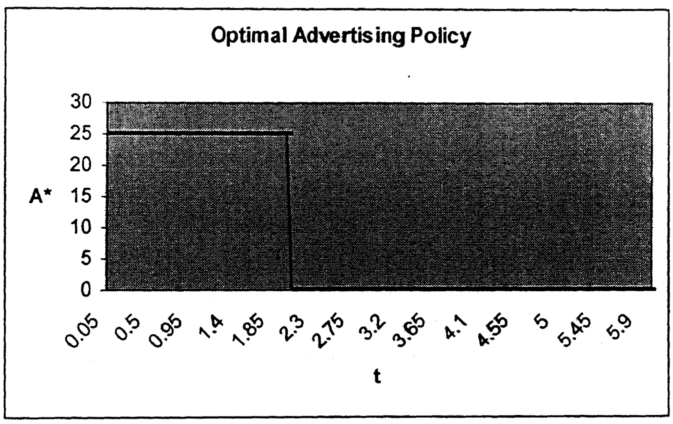


FIGURE 6.3(b). Plot of Optimal Advertising Policy.

For both the cases above, the optimal advertising policy is to gradually decrease the advertising expenditure rate and the explanation for this is similar to that given in the

previous numerical example (*refer 7(B)*). Thus the effect of length of the planning period is only to allow for further adoptions to take place and once a desirable level of awareness has reached, advertising may be stopped completely or maintained at a low level if forgetting by the consumers is expected.

6.4 Effect of Profit Margin per unit (g_X, g_Y):

In this numerical example setting we will compare the optimal advertising policies for the following two cases. In the first case, the gross profit margin exclusive of advertising for unit product sale through primary channel (g_X) is taken to be greater than that through secondary channel (g_Y). The second case where ($g_Y > g_X$), is further broken into two cases where the ratio (g_Y/g_X) is taken to be different. All the other model parameters have the same values for all the cases.

(a) $g_X > g_Y$:

We choose the following parameters-

N_X	N_Y	r	g_Y	g_X	β	b	δ	I(0)	ε	Y(0)	k_1	k_2	k_3	k_4	t_o	T
75	50	0	7	10	0.04	0.327	0	0.01	0	0	0.5	0.1	0.4	0.5746	2	6

Iterating the values of the variables for $n = 120$ time periods, therefore $\Delta t = T/120 = 0.05$ we obtain the following optimal advertising policy (*refer Table B.4.1*).

A1	A2	A3	A4
25	25	25	0

I	Y	X_1	X_2	$X=X_1+X_2$	X+Y	π
0.747	10.106	18.081	14.461	32.542	42.648	299.434

The optimal advertising policy (25, 25, 25, 0) is depicted in Figure 6.4(a).

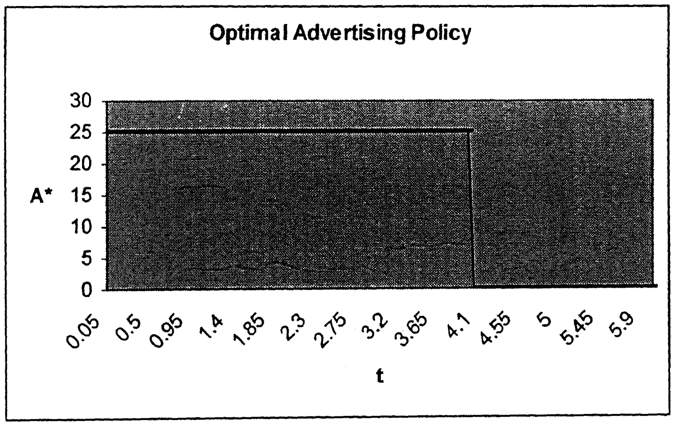


FIGURE 6.4(a). Plot of Optimal Advertising Policy.

(b) $g_Y > g_X$:

We choose the following parameters-

N_X	N_Y	r	g_Y	g_X	β	b	δ	I(0)	ε	Y(0)	k_1	k_2	k_3	k_4	t_o	T
75	50	0	10	5	0.04	0.327	0	0.01	0	0	0.5	0.1	0.4	0.5746	2	6

Iterating the values of the variables for $n = 120$ time periods, therefore $\Delta t = T / 120 = 0.05$ we obtain the following optimal advertising policy (*refer Table B.4.2*).

A1	A2	A3	A4
25	25	0	0

I	Y	X_1	X_2	$X=X_1+X_2$	X+Y	π
0.621	9.040	13.818	11.652	25.470	34.510	169.410

The optimal advertising policy (25, 25, 0, 0) is depicted in Figure 6.4(b).

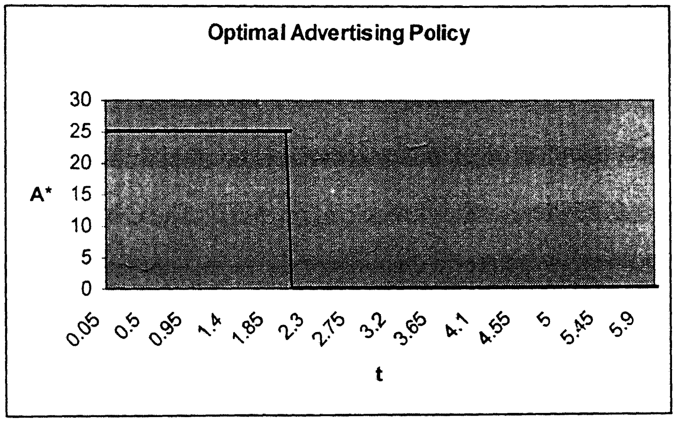


FIGURE 6.4(b). Plot of Optimal Advertising Policy.

(c) $g_Y > g_X$:

We choose the following parameters-

N_X	N_Y	r	g_Y	g_X	β	b	δ	I(0)	ε	Y(0)	k_1	k_2	k_3	k_4	t_o	T
75	50	0	10	7	0.04	0.327	0	0.01	0	0	0.5	0.1	0.4	0.5746	2	6

Iterating the values of the variables for $n = 120$ time periods, therefore $\Delta t = T / 120 = 0.05$ we obtain the following optimal advertising policy (*refer Table B.4.3*).

A1	A2	A3	A4
25	25	25	0

I	Y	X_1	X_2	$X=X_1+X_2$	X+Y	π
0.747	10.106	18.081	14.461	32.542	42.648	231.313

The optimal advertising policy (25, 25, 25, 0) is depicted in Figure 6.4(c).

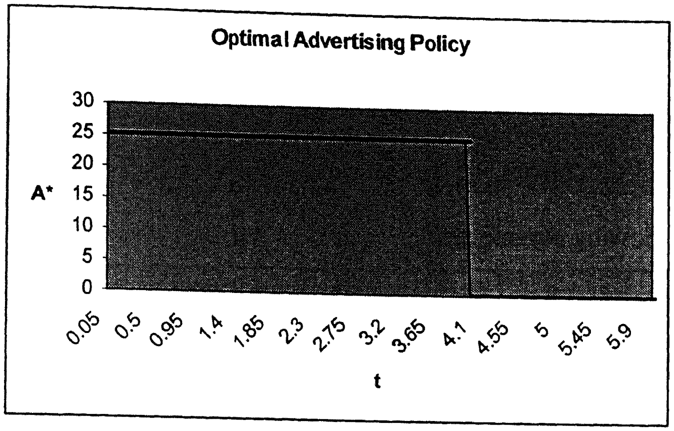


FIGURE 6.4(c). **Plot of Optimal Advertising Policy.**

Considering case (a), the optimal advertising rate is steady before opening of the market (25, 25) but falls significantly after market opening (25, 0). This can be explained by Proposition 2 (*refer 6.2.2*). Since ($g_X/g_Y > 1$), hence it is optimal to have $\dot{A}(t_o^+) < \dot{A}(t_o^-)$, that is, to decrease advertising rate significantly after market opening.

Although both in case (b) and (c), ($g_X/g_Y < 1$) but still there is difference in the optimal advertising policies. This can also be explained by Proposition 2 (*refer 6.2.2*). It states that (g_X/g_Y) should be less than a critical value so that it is optimal to have $\dot{A}(t_o^+) > \dot{A}(t_o^-)$.

For case (b), $g_X/g_Y = 0.5$ whereas for case (c), $g_X/g_Y = 0.7$. The critical value can be calculated by substituting the numerical values of the model parameter in (6.2.2.7).

For the current example the critical value is:

$$\frac{k_1 - k_2}{(k_3 - k_4) + \dfrac{N_X}{N_Y} k_4} = \frac{0.5 - 0.1}{(0.4 - 0.5746) + \dfrac{75}{50} 0.5746} = 0.582$$

Since for case (b), the inequality (6.2.2.7) is satisfied hence it is optimal to have $\dot{A}(t_o^+) > \dot{A}(t_o^-)$, that is, to keep steady the advertising expenditure rate after market opening. Hence the optimal advertising policy for case (b) is (25, 25, 0, 0).

However, case (c) does not satisfy the inequality (6.2.2.7), therefore the optimal advertising policy is different from case (b).

6.5 Effect of Likelihood of Adoption (k_1, k_2, k_3, k_4):

In this numerical example setting we will compare the optimal advertising policies for the following two cases. In the first case, the likelihood of adoption through secondary

channel before opening of the market (k_1) is taken to be greater than that after market opening (k_2). In the second case k_1 is given a value less than k_2. All the other model parameters have the same values for both the cases.

(a) $k_1 > k_2$:

We choose the following parameters-

N_X	N_Y	r	g_Y	g_X	β	b	δ	I(0)	ε	Y(0)	k_1	k_2	k_3	k_4	t_o	T
75	50	0	10	5	0.04	0.327	0	0.01	0	0	0.5	0.1	0.4	0.5746	2	6

Iterating the values of the variables for $n = 120$ time periods, therefore $\Delta t = T/120 = 0.05$ we obtain the following optimal advertising policy (*refer Table B.5.1*).

A1	A2	A3	A4		I	Y	X_1	X_2	$X=X_1+X_2$	X+Y	π
25	25	0	0		0.621	9.040	13.818	11.652	25.470	34.510	169.410

The optimal advertising policy (25, 25, 0, 0) is depicted in Figure 6.5(a).

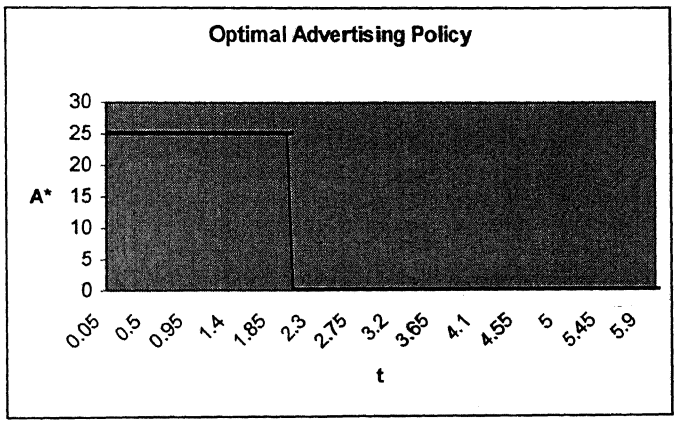


FIGURE 6.5(a). Plot of Optimal Advertising Policy.

(b) $k_1 < k_2$:

We choose the following parameters-

N_X	N_Y	r	g_Y	g_X	β	b	δ	I(0)	ε	Y(0)	k_1	k_2	k_3	k_4	t_o	T
75	50	0	10	5	0.04	0.327	0	0.01	0	0	0.5	0.6	0.4	0.5746	2	6

Iterating the values of the variables for $n = 120$ time periods, therefore $\Delta t = T/120 = 0.05$ we obtain the following optimal advertising policy (*refer Table B5.2*).

A1	A2	A3	A4		I	Y	X_1	X_2	$X=X_1+X_2$	X+Y	π
25	25	25	0		0.747	22.156	11.209	14.461	25.670	47.827	251.979

The optimal advertising policy (25, 25, 25, 0) is depicted in Figure 6.5(b).

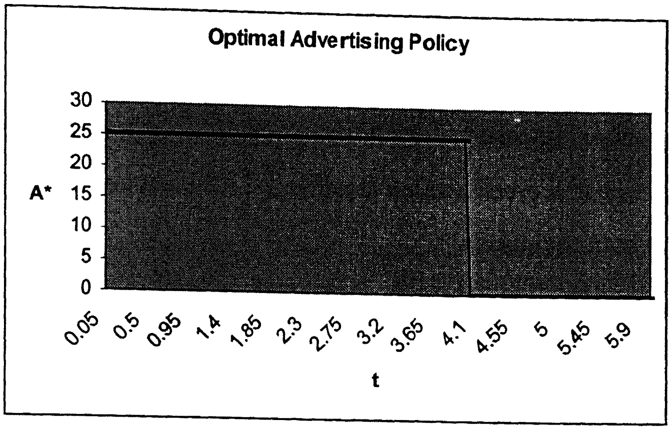


FIGURE 6.5(b). Plot of Optimal Advertising Policy.

The explanation for the difference in optimal advertising policy for the two cases is similar to that given in the previous numerical example. However, here ($g_X/g_Y = 0.5$) is the same for the two cases. The difference lies in the critical value which is different for the two cases due to the difference in the numerical values of the likelihood of adoption (k_1 and k_2).

For case (a), the critical value is:

$$\frac{k_1 - k_2}{(k_3 - k_4) + \frac{N_X}{N_Y} k_4} = \frac{0.5 - 0.1}{(0.4 - 0.5746) + \frac{75}{50} 0.5746} = 0.582$$

The inequality (6.2.2.7) is satisfied and the explanation for case (a) is same as that discussed in the previous example.

For case (b), the critical value is:

$$\frac{k_1 - k_2}{(k_3 - k_4) + \frac{N_X}{N_Y} k_4} = \frac{0.5 - 0.6}{(0.4 - 0.5746) + \frac{75}{50} 0.5746} = -0.145$$

Since for case (b), the critical value comes out to be negative but the ratio (g_X/g_Y) will always be positive. Hence, (g_X/g_Y) can never be less than this critical value and the inequality (6.2.2.7) is never satisfied.

Chapter 7

Managerial Implications

A firm planning to introduce a new consumer durable in an emerging market which is under regulatory restrictions, will have to decide the advertising policy to follow which will maximize the net discounted profits at the end of planning horizon. The firm as well the consumers in that market are aware of the tentative market opening time, after which the restrictions imposed by the government will be lifted and the global companies will be allowed to launch their products in the market. Although the product is an innovation in the unexplored market, the firm is well aware of the fact that the first mover advantage it will enjoy at the start will not last for long before they start facing competition through similar products launched by other companies which are late entrants in the market. The product will have its life cycle, at the end of which it would be appropriate for the firm to withdraw the product from the market because of a very less demand existing for it. This may be due to either obsolescence of the product technology or a low remaining potential adopter population. Due to the limited life of the product in the market, the firm will want to have maximum adoptions possible by the potential adopter population before the end of the planning horizon. Thus, it is imperative for the firm to advertise about the product before its launch as well after so as to inform the potential adopter population about the availability of a product with its various features.

The results obtained for optimal advertising policy using optimal control methodology under various market situations and conditions are summarized in Table 7.1. It is proposed that the firm should advertise before opening of the market which is also assumed to be the time at which the product is launched in the market. The optimal advertising policy is to have a high advertising expenditure rate in the beginning and gradually decrease it as time approaches the market opening time for the situation where discount rate and advertising is assumed to be negligible. This is consistent with previous studies, for example, Kalish (1985). Further, the profit margin for unit product sale exclusive of advertising through secondary channel should be greater than that through primary channel and consumer anticipation of better deal after market opening is not considered. These assumptions are not strict and for insignificant level of the above mentioned effects, the optimal advertising policy will remain the same. If the product marketed by the firm is characterized by very few

product features, then forgetting by the consumers can be assumed to be negligible. This is because 'relearning' of the product features may not be required and hence it justifies the decrease in advertising with time. For a market situation in which the discount rate assumed to be very low, ignoring discount rate from the model will not affect the results significantly. However it was earlier shown in a numerical example that abnormally high value of discount rate and forgetting rate may reverse the optimal advertising policy, that is, it would be optimal to increase advertising expenditure rate with time. Since when the rewards and expenditures are discounted back to time zero, expenditure done on advertising at a later period of time will have less monetary value as compared to what it is done initially, hence it will contribute to the net discounted profits at the end of the planning period.

The firm should be aware that its pricing policy for the product also affects the optimal advertising policy significantly as shown in Table7.1.The numerical value of the profit margin through the two channels will determine the advertising policy before and around the market opening time. It is shown that if the profit margin through secondary channel is set greater than that through primary channel and other mentioned conditions are satisfied, then it will always be optimal to have a monotonic decreasing advertising rate. This can be expected reasonably in most global marketing conditions. Also, if the ratio of the profit margin through the primary channel to that of secondary channel is less than or greater than some critical value, then the optimal policy will be different for the two cases. In the first case, in which the ratio is less than the critical value, it is optimal to the decrease advertising expenditure at a lower rate after market opening as compared to that before opening of market. The implication of this result is that since the overall advertising expenditure rate decreases with time, it is better to do most of the advertising before opening of the market, and then after market opening, the advertising expenditure rate be maintained at a lower level. Similarly, for the other case, in which the ratio is greater than a critical value, it is optimal to the decrease advertising expenditure at a higher rate after market opening as compared to that before opening of market. This result implies that advertising should be continued after opening of the market and advertising expenditure rate should be significant initially during the market opening time after which it should be drastically reduced on nearing the end of planning horizon.

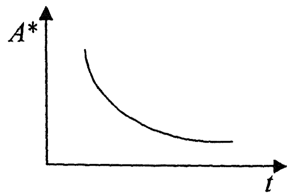
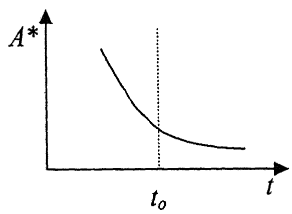
The explanation for the above contrasting results is as follows. For the situation in which the profit margin through primary channel is less than through secondary channel, it will be optimal to advertise significantly before market opening which will result in higher product adoption through secondary channel. This is because the firm gets greater profit by

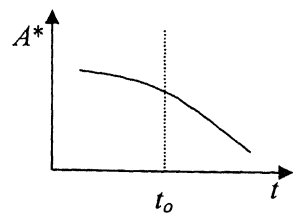
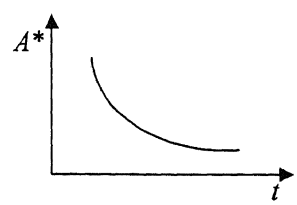
product adoption through secondary channel as compared to that through primary channel. Similarly, for the other situation, in which it is profitable for the firm to have product adopted through primary channel to that through secondary channel, the firm should advertise around the market opening time and even after that for some time before finally stop advertising near the end of the planning horizon.

The optimal advertising policy after opening of the market is always to decrease the advertising rate monotonically for the situation in which discount rate and forgetting rate is assumed to be zero. It is interesting to note that pricing policy has no effect on optimal advertising policy after market opening. This is due to the fact that the purpose of advertising is to make the consumers aware of the product and motivate them to adopt it. Since by the end of the planning horizon, the awareness level may be assumed to reach close to unity hence further advertising might not be required. Advertising expenditure rate may be maintained at a low level to counter the effect of forgetting and help in 'relearning' by the consumers.

Finally, from all the results discussed so far it is evident that optimal advertising policy is to have a monotonic decreasing advertising expenditure rate for most of the practical situations which a firm might face in entering an emerging market under regulatory restrictions. Apart from some extreme situations, for example in case of high discounting rate and forgetting rate where the optimal advertising policy would be to increase the advertising expenditure rate, advertising should decrease with time.

Table 7.1: Summary of Results for Optimal Advertising Policy

	Conditions	Optimal Advertising Policy	Normative Recommendations
$(t < t_o)$	$r \approx 0$, $\delta \approx 0$, $\theta(t) \approx 1$, $g_Y > g_X$.		The advertising expenditure rate (A) should be gradually decreased with time before market opening if the pricing policy set by the firm is to have profit margin for unit product sale through secondary channel greater than that through primary channel and the given conditions are satisfied.
$(t \approx t_o)$	$\frac{g_X}{g_Y} < \frac{(\theta(t_o)k_1 - k_2)}{(k_3 - k_4) + \frac{N_X}{N_Y} k_4}$		The Advertising expenditure rate (A) should decrease at a lower rate after market opening (t_o) as compared to that before market opening if the ratio of profit margin for unit product sale through primary channel to that through secondary channel is less than the critical value given by the inequality.

<table>
<tr>
<td></td>
<td>$$\frac{g_X}{g_Y} > \frac{\theta(t_o)k_1k_3 - r(k_2 - \theta(t_o)k_1)}{k_3[r + \theta(t_o)k_1]}$$</td>
<td>
</td>
<td>The Advertising expenditure rate (A) should be decreased at a higher rate after market opening (t_o) as compared to that before market opening if the ratio of profit margin for unit product sale through primary channel to that through secondary channel is more than a critical value given by the inequality.</td>
</tr>
<tr>
<td>$(t > t_o)$</td>
<td>$r \approx 0,$
$\delta \approx 0.$</td>
<td>
</td>
<td>The advertising expenditure rate (A) should be gradually decreased with time after market opening if the given conditions are satisfied.</td>
</tr>
</table>

Chapter 8

Conclusions, Limitations and Future Directions

8.1 Conclusions

In this research, a modeling framework is developed for characterizing new product diffusion in a market under regulatory restrictions. A firm with a global presence, targets this emerging market in order to expand its market for the new product which has already been available in the foreign market for some time. Since the product is new to this market, and hence an innovation, so the firm intends to be the pioneer in the market and enjoy first mover advantages. Before the regulatory restrictions are lifted and the product is available in the local market, the only channel of adoption possible during that time is to adopt it from the foreign market. This channel of adoption is termed as "secondary channel" in this research, and comprises of relatives and acquaintances in the foreign market that a consumer may have. After the regulations are removed, the firm will be allowed to launch its product in the market. The channel of adoption of the product from the local market is termed as "primary channel". Even after the primary channel comes into existence, some adoptions might still take place through the secondary channel. The firm may choose to set different price and quality levels for the same or similar product available in the two markets. The product features available in the foreign market and the local market may also differ. The consumers who have access to secondary channel will have to decide the channel of adoption based on comparing the perceived overall value they will get by adopting the product through a particular channel.

The model proposed in this research is built on the Kalish model (1985) with some modifications to account for the market situation described above. Awareness diffusion is separated from product adoption, which is imperative for the situation addressed by the current research. However, uncertainty regarding product performance is treated differently and is incorporated in the model to account for consumer's reluctance to adopt the new product from the local market. Uncertainty is not associated with the product available in the foreign market since it has been available there for some time. It is tested and information about its performance is assumed to be known to the consumers. Another effect of significance before opening of the market in the market situation discussed is of consumer anticipation about a better product available at lower price in the local market which will have the effect of delaying product adoption through secondary channel. Also, forgetting by

consumers is considered and it is assumed that a consumer adopts the product only on having full information about it. Thus, forgetting effect results in decrease in effective awareness level and hence the necessity for relearning. All the effects discussed so far has been incorporated in the model to closely reflect the market conditions which the firm might face.

The problem dealt by this research is to find the optimal advertising policy which the firm should follow in order to maximize its net discounted profits at the end of the planning horizon. We show how advertising helps in awareness diffusion by providing information about the product features and availability to the potential adopters. Thus it helps in product adoption by making the product adoption curve to peak earlier. But advertising involves cost which is measured by the advertising expenditure rate. The model developed in this research is used to determine the optimal advertising policy by applying optimal control methodology to the proposed product diffusion model. The results obtained may be summarized as follows. The optimal normative advertising policy recommends a monotonic decreasing advertising expenditure rate over time for most practical situations specifically after opening of the market, in which discounting rate and forgetting rate can be assumed to be negligible. Further, if the profit margin for unit product sale through secondary channel is greater than that through primary channel, then the optimal policy before market opening will always be monotonic decreasing. In the event of extreme conditions (where the value of discounting rate and forgetting rate are significant), the optimal advertising policy would be to increase the advertising expenditure rate. It is also shown, how pricing policy set by the firm for the product in the two markets will determine the optimal advertising policy around the time of opening of the market.

8.2 Limitations

The model developed in this research is built on the Kalish model (1985). Therefore, the proposed model inherits the limitations from it. First, the market potential is assumed to be constant over time for the whole planning period. Since optimal pricing policies have not been considered in this research, the dependence of market potential on the product price has not been explicitly modeled. Further, Kalish (1985) has considered uncertainty associated with product performance to affect the market potential whereas in the current research, uncertainty is assumed to affect the remaining potential adopter population which has not adopted the product by that time.

The model is developed on the assumption that the firm is pioneer in the emerging market and hence the model is best applicable to situations of a monopoly product. Also, we

assume that the adopter makes single purchase of the product. The population is assumed to be homogeneous with respect to awareness diffusion and their likelihood of product adoption. The effect of difference in the effectiveness of information transmission through word of mouth by actual adopters in comparison to that by just informed individuals has not been considered in the model as done by Kalish (1985). This assumption was made in order to have further analysis of the model tractable for implementing optimal control technique. Also uncertainty effect has not been considered in the optimal control model for the same reason mentioned above. In the optimal control model, profit margin for unit product sale is assumed to be constant over time which may not be the case.

8.3 Directions for Future Research

One of the major directions for future research would be the introduction of competitive effects in the present modeling framework. Further, repeat purchases may be incorporated in the model. Also, optimal pricing policy may be derived along with optimal advertising policy and further insights may be gained on how they affect each other as it was evident in this research that pricing policy affects the optimal advertising policy. The effect of uncertainty on the optimal advertising policy may also be studied which was neglected in this research. Profit margins may be taken to vary with the number of adoptions to take into account reduction in variable costs due to experience effect. Finally, the problem formulated in this research is a deterministic case in which the market opening time is assumed to be known exactly in advance by the consumers and the firm. However, a more practical situation would be one in which the market opening time is not known for sure and there is an element of uncertainty associated with it.

References

1. Arnold, D. and Quelch, J. (1998),""New strategies in emerging economies," *Sloan Management Review*, 40(l), pp. 7-20.
2. Bass, Frank M. (1969), "A new product growth model for consumer durables," *Management Science*, Vol.15 (January), pp. 215-27.
3. Belderbos, R. and Sleuwaegen, L. (1996), "Japanese firms and the decision to invest abroad: Business groups and regional core networks," *Review of Economics and Statistics*, 78, pp. 214-220.
4. Bhattacharyya, S.K. and Rahman, Z. (2003), "Sources of first mover advantages in emerging markets - an Indian perspective," *European Business Review, Bradford*, Vol. 15:6, pp. 361-362.
5. Dockner, E. and S. Jorgensen (1988a), "Optimal Advertising Policies for Diffusion Models of New Product Innovations in Monopolistic Situation", *Management Science*, vol. 34, pp.119-130.
6. Dolan, R. and A. Jeuland (1981), "Experience Curves and Dynamic Demand Models: Implications for Optimal Pricing Strategies", *Journal of Marketing Research*, vol. 45, pp. 52-62.
7. Fourt, L.A. and J.W. Woodlock (1960), "Early Prediction of Market Success for New Grocery Products", *Journal of Marketing*, vol. 25, pp. 31-38.
8. Gilbert, R. and Train, T., "A qualitative choice analysis of technology adoption by electric utilities", *Final Report to EPRI*, RP1289-1-1, 1981.
9. Hill, J. (1984), "Targeting promotions in lesser-developed countries: a study of multinational corporation strategies," *Journal of Advertising*, Vol. 13 No. 4, pp. 39-48.
10. Horsky, D. and L. Simon (1983), "Advertising and the Diffusion of New Product," *Marketing Science*, vol. 1, pp. 1-18.
11. Horsky, Dan and Leonard S. Simon (1983), "Advertising and the diffusion of new products," *Management Science*, 1 (winter), pp. 1-18.
12. Kalish, Sholmo (1983), "Monopolist pricing with dynamic demand and production cost," *Marketing Science*, 2 (spring), pp. 135-160.
13. Kalish, Sholmo (1985), "A new product adoption model with pricing, advertising and uncertainty," *Management Science*, 31 (December), pp. 1569-1585.

14. Kamien, M. and N. Schwartz, (1981). *Dynamic optimization: The calculus of variations and optimal control theory in economics and management*, Amsterdam: North-Holland.

15. Khanna, T., and K. Palepu (1997), "Why focused strategies may be wrong for emerging markets," *Harvard Business Review*, 75(4), pp. 41-51.

16. Lee, Y. and O'Connor, G.C. (2003), "The impact of communication strategy on launching new products: The moderating role of product innovativeness," *The Journal of Product Innovation Management*, 20, pp. 4-21.

17. Mahajan, V. and E. Muller (1979), "Innovation diffusion and new-product growth models in marketing," *Journal of Marketing*, 43, pp. 55-68.

18. Mahajan, V. and R. Peterson (1978), "Innovation diffusion in a dynamic potential adopter population," *Management Science*, Vol. 24, pp. 1589-1597.

19. Mahajan, V., and R.A. Peterson (1978), "Innovation diffusion in a dynamic potential adopter population," *Management Science*, 24, pp. 1589-1597.

20. Mahajan, V., E. Muller and R.A. Kerin (1984), Introduction strategy for new products with positive and negative word-of-mouth," *Management Science*, 30, pp. 1389-1404.

21. Mahajan, Vijay, Muller, Eitan, and Bass, Frank M. (1990). "New Product Diffusion Models in Marketing: A Review and Directions for Research," *Journal of Marketing*, Vol. 54 (January), pp. 1-26.

22. Mansfield, E. (1961), "Technical Change and the Rate of Imitation", *Econometrica*, Vol. 29, pp. 741-766.

23. Mesak, H. and W. Berg (1995), "Incorporating, Price and Replacement Purchases in New Products Diffusion Models for Consumer Durables," *Decision Sciences*, Vol. 26, pp. 425-449.

24. Miller, D. and Friesen, P. (1983) "The Correlates of Entrepreneurship in Three Types of Firms," *Management Science*, 29, pp. 770–791.

25. Mittal, Sharad, and Sanjeev Swami (2004), "Pioneering Advantage: Conceptualization, Proposition, and Analysis in Indian Context," *Vikalpa: The Journal of Decision Makers*, Volume 29, Number 3, July-September, pp. 15 — 33.

26. Nakata, Cheryl and K. Sivakumar (1997), "Emerging market conditions and their impact on first mover advantages," *International Marketing Review*, Vol. 14 No. 6, pp. 461-484.

27. Oren, Shmuel S. and Rick G. Schwartz (1988), "Diffusion of new products in risk-sensitive markets," *Journal of Forecasting*, 7, pp. 273-287.

28. Robinson, B. and C. Lakhani (1975), "Dynamic price models for new product planning," *Management Science*, Vol. 10, pp. 1113-1122.
29. Robinson, W.T. (1985), "Sources of market pioneer advantages: the case of industrial goods industries", *Journal of Marketing Research*, February, p.87.
30. Robinson, W.T. and Fornell, C. (1985), "Sources of market pioneer advantage in consumer goods industries," *Journal of Marketing Research*, August, p.305.
31. Rogers, E.M. and Shoemaker, F.F. (1971), *Communication of Innovations*, The Free Press, New York.
32. Rogers, E.M. (1983), *Diffusion of Innovation*, The Free Press, New York, NY.
33. Sazanami, Y. (1996), "Globalization and regionalization: Japanese multinational enterprises in the Asia-Pacific," *Journal of Asian Economics*, 8, pp. 1-13.
34. Swami, Sanjeev and Pankaj Khairnar, "Optimal Normative Policies for Marketing of Products with Limited Availability," (*submitted to Annals of Operations Research*).
35. Teng, J. and R. Thompson (1983), "Oligopoly models for optimal advertising when production costs obey a learning curve," *Management Science*, Vol. 29, pp. 1087-1101.

1. For $0 \le t < t_o$,

$$\frac{\partial H_1}{\partial A} = 0 .$$

Substituting expression for H_I,

$$\frac{\partial}{\partial A}\left[-A + (g_Y + m_{1Y})\dot{Y} + m_{1I}\dot{I}\right] = 0$$

Since $\dfrac{\partial \dot{Y}}{\partial A} = 0$.

$$\therefore -1 + m_{1I}\frac{\partial \dot{I}}{\partial A} = 0$$

$$\Rightarrow m_{1I} = \frac{1}{\left(\dfrac{\partial \dot{I}}{\partial A}\right)}$$

But,

$$\frac{\partial \dot{I}}{\partial A} = (1-I)\frac{\partial}{\partial A}[f(A) + bI] + [f(A) + bI]\frac{\partial}{\partial A}(1-I) - \delta\frac{\partial I}{\partial A}$$

$$\Rightarrow \frac{\partial \dot{I}}{\partial A} = (1-I)\left[f'(A) + b\frac{\partial I}{\partial A}\right] - [f(A) + bI]\frac{\partial I}{\partial A} - \delta\frac{\partial I}{\partial A}$$

Since $\dfrac{\partial I}{\partial A} = 0$

$$\therefore \frac{\partial \dot{I}}{\partial A} = (1-I)f'(A)$$

Therefore,

$$m_{1I} = \frac{1}{(1-I)f'(A)}$$

$$\Rightarrow \frac{dm_{1I}}{dt} = -\frac{1}{[(1-I)f'(A)]^2}\frac{d}{dt}[(1-I)f'(A)]$$

Now,

$$\frac{d}{dt}[(1-I)f'(A)] = (1-I)\frac{d}{dt}f'(A) + f'(A)\frac{d}{dt}(1-I)$$

$$= (1-I)f''(A)\dot{A} - f'(A)\dot{I}$$

$$\therefore \dot{m}_{1I} = -\frac{(1-I)f''(A)\dot{A} - \dot{I}f'(A)}{[(1-I)f'(A)]^2} \tag{A.1.1}$$

Also $\dot{m}_{1I} = rm_{1I} - \frac{\partial H_1}{\partial I}$

Now,

$$\frac{\partial H_1}{\partial I} = \frac{\partial}{\partial I}\left[-A + (g_Y + m_{1Y})\dot{Y} + m_{1I}\dot{I}\right]$$

$$= (g_Y + m_{1Y})\frac{\partial \dot{Y}}{\partial I} + m_{1I}\frac{\partial \dot{I}}{\partial I}$$

Since $\frac{\partial A}{\partial I} = 0$

$$\Rightarrow \dot{m}_{1I} = \left(r - \frac{\partial \dot{I}}{\partial I}\right)m_{1I} - (g_Y + m_{1Y})\frac{\partial \dot{Y}}{\partial I} \tag{A.1.2}$$

From Equation (A.1.1) and (A.1.2),

$$-\frac{(1-I)f''(A)\dot{A} - \dot{I}f'(A)}{[(1-I)f'(A)]^2} = \left(r - \frac{\partial \dot{I}}{\partial I}\right)m_{1I} - (g_Y + m_{1Y})\frac{\partial \dot{Y}}{\partial I}$$

$$\Rightarrow -\left[\frac{f''(A)}{f'(A)}\dot{A} - \frac{\dot{I}}{1-I}\right]\frac{1}{(1-I)f'(A)} = \left(r - \frac{\partial \dot{I}}{\partial I}\right)\frac{1}{(1-I)f'(A)} - (g_Y + m_{1Y})\frac{\partial \dot{Y}}{\partial I}$$

$$\Rightarrow -\left[\frac{f''(A)}{f'(A)}\dot{A} - \frac{\dot{I}}{1-I}\right] = \left(r - \frac{\partial \dot{I}}{\partial I}\right) - (g_Y + m_{1Y})\frac{\partial \dot{Y}}{\partial I}(1-I)f'(A)$$

Or,

$$-\frac{f''(A)}{f'(A)}\dot{A} = r - \frac{\partial \dot{I}}{\partial I} - \frac{\dot{I}}{1-I} - (g_Y + m_{1Y})\frac{\partial \dot{Y}}{\partial I}(1-I)f'(A) \tag{A.1.3}$$

Now,

$$\frac{\partial \dot{I}}{\partial I} = (1-I)\frac{\partial}{\partial I}[f(A) + bI] + [f(A) + bI]\frac{\partial}{\partial I}(1-I) - \delta$$

$$\Rightarrow \frac{\partial \dot{I}}{\partial I} = (1-I)b - [f(A) + bI] - \delta \tag{A.1.4}$$

And,

$$\frac{\partial \dot{Y}}{\partial I} = \theta(t)k_1 \frac{\partial}{\partial I}[N_Y I - Y]$$

$$\Rightarrow \frac{\partial \dot{Y}}{\partial I} = N_Y \theta(t) k_1 \qquad \text{(A.1.5)}$$

Substituting from Equation (A.1.4) and (A.1.5) into (A.1.3) and simplifying,

$$-\frac{f''(A)}{f'(A)}\dot{A} = \left[r + \frac{\delta}{1-I}\right] - (1-I)[b + (g_Y + m_{1Y})N_Y \theta(t) k_1 f'(A)]$$

Where m_{IY} is given by solving the following first-order ODE:

$$\dot{m}_{1Y} = rm_{1Y} - \frac{\partial H_1}{\partial Y}$$

Now,

$$\frac{\partial H_1}{\partial Y} = \frac{\partial}{\partial Y}\left[-A + (g_Y + m_{1Y})\dot{Y} + m_{1I}\dot{I}\right]$$

$$= (g_Y + m_{1Y})\frac{\partial \dot{Y}}{\partial Y} + m_{1I}\frac{\partial \dot{I}}{\partial Y}$$

Since $\frac{\partial A}{\partial Y} = 0$ and $\frac{\partial \dot{I}}{\partial Y} = 0$,

$$\therefore \dot{m}_{1Y} = \left(r - \frac{\partial \dot{Y}}{\partial Y}\right)m_{1Y} - g_Y \frac{\partial \dot{Y}}{\partial Y}$$

But,

$$\frac{\partial \dot{Y}}{\partial I} = \theta(t)k_1 \frac{\partial}{\partial Y}[N_Y I - Y]$$

$$\Rightarrow \frac{\partial \dot{Y}}{\partial Y} = -\theta(t)k_1$$

$$\therefore \dot{m}_{1Y} = [r + \theta(t)k_1]m_{1Y} + g_Y \theta(t) k_1$$

On solving the above first-order ODE,

$$\frac{d}{dt}\left\{m_{1Y}e^{-\int[r+\theta(t)k_1]dt}\right\} = g_Y \theta(t) k_1 e^{-\int[r+\theta(t)k_1]dt}$$

Or,

$$\left[m_{1Y}e^{-rt-k_1\int\theta(t)dt}\right]_t^{t_o} = g_Y k_1 \int_t^{t_o}\left[\theta(t)e^{-rt-k_1\int\theta(t)dt}\right]dt$$

Assuming $\theta(t)=1$,

$$\left[m_{1Y}e^{-rt-k_1t}\right]_t^{t_o} = g_Y k_1 \int_t^{t_o}\left[e^{-rt-k_1t}\right]dt$$

$$\Rightarrow m_{1Y}(t_o)e^{-(r+k_1)t_o} - m_{1Y}(t)e^{-(r+k_1)t} = -\frac{g_Y k_1}{r+k_1}\left[e^{-(r+k_1)t_o} - e^{-(r+k_1)t}\right]$$

$$\therefore m_{1Y} = m_{1Y}(t_o)e^{-(r+k_1)(t_o-t)} + \frac{g_Y k_1}{r+k_1}\left[e^{-(r+k_1)(t_o-t)} - 1\right]$$

2. For $t_o < t \le T$,

$$\frac{\partial H_2}{\partial A} = 0$$

Substituting expression for H_2,

$$\frac{\partial}{\partial A}\left[-A+\left(g_X+m_{2X_1}\right)\dot{X}_1+\left(g_X+m_{2X_2}\right)\dot{X}_2+\left(g_Y+m_{2Y}\right)\dot{Y}+m_{2I}\dot{I}\right]=0$$

$$\Rightarrow -1+\left(g_X+m_{2X_1}\right)\frac{\partial \dot{X}_1}{\partial A}+\left(g_X+m_{2X_2}\right)\frac{\partial \dot{X}_2}{\partial A}+\left(g_Y+m_{2Y}\right)\frac{\partial \dot{Y}}{\partial A}+m_{2I}\frac{\partial \dot{I}}{\partial A}=0$$

Since $\frac{\partial \dot{X}_1}{\partial A}=0, \frac{\partial \dot{X}_2}{\partial A}=0, \frac{\partial \dot{Y}}{\partial A}=0$

$$\therefore -1+m_{2I}\frac{\partial \dot{I}}{\partial A}=0$$

Or,

$$m_{2I} = \frac{1}{\left(\dfrac{\partial \dot{I}}{\partial A}\right)}$$

But,

$$\frac{\partial \dot{I}}{\partial A} = (1-I)\frac{\partial}{\partial A}\left[f(A)+bI\right]+\left[f(A)+bI\right]\frac{\partial}{\partial A}(1-I)-\delta\frac{\partial I}{\partial A}$$

$$\Rightarrow \frac{\partial \dot{I}}{\partial A} = (1-I)\left[f'(A)+b\frac{\partial I}{\partial A}\right]-\left[f(A)+bI\right]\frac{\partial I}{\partial A}-\delta\frac{\partial I}{\partial A}$$

Since $\frac{\partial I}{\partial A}=0$

$$\therefore \frac{\partial \dot{I}}{\partial A} = (1-I)f'(A)$$

Therefore,

$$m_{2I} = \frac{1}{(1-I)f'(A)}$$

$$\Rightarrow \frac{dm_{2I}}{dt} = -\frac{1}{[(1-I)f'(A)]^2}\frac{d}{dt}[(1-I)f'(A)]$$

Now,

$$\frac{d}{dt}[(1-I)f'(A)] = (1-I)\frac{d}{dt}f'(A) + f'(A)\frac{d}{dt}(1-I)$$

$$= (1-I)f''(A)\dot{A} - f'(A)\dot{I}$$

$$\Rightarrow \dot{m}_{2I} = -\frac{(1-I)f''(A)\dot{A} - \dot{I}f'(A)}{[(1-I)f'(A)]^2}$$

But,

$$-\frac{(1-I)f''(A)\dot{A} - \dot{I}f'(A)}{[(1-I)f'(A)]^2} = -\left[\frac{f''(A)}{f'(A)}\dot{A} - \frac{\dot{I}}{1-I}\right]\frac{1}{(1-I)f'(A)}$$

$$\therefore \dot{m}_{2I} = -\left[\frac{f''(A)}{f'(A)}\dot{A} - \frac{\dot{I}}{1-I}\right]\frac{1}{(1-I)f'(A)} \qquad \text{(A.2.1)}$$

Where m_{2I} is given by solving the following first-order ODE:

$$\dot{m}_{2I} = rm_{2I} - \frac{\partial H_2}{\partial I}$$

Now,

$$\frac{\partial H_2}{\partial I} = \frac{\partial}{\partial I}\left[-A + (g_X + m_{2X_1})\dot{X}_1 + (g_X + m_{2X_2})\dot{X}_2 + (g_Y + m_{2Y})\dot{Y} + m_{2I}\dot{I}\right]$$

$$= (g_X + m_{2X_1})\frac{\partial \dot{X}_1}{\partial I} + (g_X + m_{2X_2})\frac{\partial \dot{X}_2}{\partial I} + (g_Y + m_{2Y})\frac{\partial \dot{Y}}{\partial I} + m_{2I}\frac{\partial \dot{I}}{\partial I}$$

Or,

$$\dot{m}_{2I} = \left(r - \frac{\partial \dot{I}}{\partial I}\right)m_{2I} - \left[(g_X + m_{2X_1})\frac{\partial \dot{X}_1}{\partial I} + (g_X + m_{2X_2})\frac{\partial \dot{X}_2}{\partial I} + (g_Y + m_{2Y})\frac{\partial \dot{Y}}{\partial I}\right] \qquad \text{(A.2.2)}$$

From Equation (A.2.1) and (A.2.2),

$$-\left[\frac{f''(A)}{f'(A)}\dot{A}-\frac{\dot{I}}{1-I}\right]=\left(r-\frac{\partial \dot{I}}{\partial I}\right)m_{2I}-\left[\left(g_X+m_{2X_1}\right)\frac{\partial \dot{X}_1}{\partial I}+\left(g_X+m_{2X_2}\right)\frac{\partial \dot{X}_2}{\partial I}+\left(g_Y+m_{2Y}\right)\frac{\partial \dot{Y}}{\partial I}\right]$$

On simplifying,

$$-\frac{f''(A)}{f'(A)}\dot{A}=r-\frac{\partial \dot{I}}{\partial I}-\frac{\dot{I}}{1-I}-\left[\left(g_X+m_{2X_1}\right)\frac{\partial \dot{X}_1}{\partial I}+\left(g_X+m_{2X_2}\right)\frac{\partial \dot{X}_2}{\partial I}+\left(g_Y+m_{2Y}\right)\frac{\partial \dot{Y}}{\partial I}\right](1-I)f'(A)$$

Now,

$$\frac{\partial \dot{I}}{\partial I}=(1-I)\frac{\partial}{\partial I}\left[f(A)+bI\right]+\left[f(A)+bI\right]\frac{\partial}{\partial I}(1-I)-\delta$$

$$\Rightarrow \frac{\partial \dot{I}}{\partial I}=(1-I)b-\left[f(A)+bI\right]-\delta \qquad \text{(A.2.3)}$$

And,

$$\frac{\partial \dot{X}_1}{\partial I}=k_3\frac{\partial}{\partial I}\left[N_Y I-(X_1+Y)\right]$$

Since $\frac{\partial X_1}{\partial I}=0, \frac{\partial Y}{\partial I}=0$.

$$\therefore \frac{\partial \dot{X}_1}{\partial I}=N_Y k_3 \qquad \text{(A.2.4)}$$

And,

$$\frac{\partial \dot{X}_2}{\partial I}=k_4\frac{\partial}{\partial I}\left[(N_X-N_Y)I-X_2\right]$$

Since $\frac{\partial X_2}{\partial I}=0$.

$$\therefore \frac{\partial \dot{X}_2}{\partial I}=(N_Y-N_Y)k_4 \qquad \text{(A.2.5)}$$

And,

$$\frac{\partial \dot{Y}}{\partial I}=k_2\frac{\partial}{\partial I}\left[N_Y I-(X_1+Y)\right]$$

Since $\frac{\partial X_1}{\partial I}=0, \frac{\partial Y}{\partial I}=0$.

$$\therefore \frac{\partial \dot{Y}}{\partial I} = N_Y k_2 \qquad \text{(A.2.6)}$$

Substituting from Equation (A.2.3), (A.2.4), (A.2.5), and (A.2.6), into (A.2.2) and simplifying,

$$-\frac{f''(A)}{f'(A)}\dot{A} = \left[r + \frac{\delta}{1-I}\right] -$$

$$(1-I)\left[b + \left\{(g_X + m_{2X_1})N_Y k_3 + (g_X + m_{2X_2})(N_X - N_Y)k_4 + (g_Y + m_{2Y})N_Y k_2\right\} f'(A)\right]$$

Where m_{2X1} is given by the multiplier equation:

$$\dot{m}_{2X_1} = r m_{2X_1} - \frac{\partial H_2}{\partial X_1}$$

Now,

$$\frac{\partial H_2}{\partial X_1} = \frac{\partial}{\partial X_1}\left[-A + (g_X + m_{2X_1})\dot{X}_1 + (g_X + m_{2X_2})\dot{X}_2 + (g_Y + m_{2Y})\dot{Y} + m_{2I}\dot{I}\right]$$

$$= (g_X + m_{2X_1})\frac{\partial \dot{X}_1}{\partial X_1} + (g_X + m_{2X_2})\frac{\partial \dot{X}_2}{\partial X_1} + (g_Y + m_{2Y})\frac{\partial \dot{Y}}{\partial X_1} + m_{2I}\frac{\partial \dot{I}}{\partial X_1}$$

Since $\frac{\partial \dot{X}_2}{\partial X_1} = 0, \frac{\partial \dot{I}}{\partial X_1} = 0$.

Therefore,

$$\frac{\partial H_2}{\partial X_1} = \left(r - \frac{\partial \dot{X}_1}{\partial X_1}\right)m_{2X_1} - \left[g_X \frac{\partial \dot{X}_1}{\partial X_1} + (g_Y + m_{2Y})\frac{\partial \dot{Y}}{\partial X_1}\right]$$

But,

$$\frac{\partial \dot{X}_1}{\partial X_1} = k_3 \frac{\partial}{\partial X_1}\left[N_Y I - (X_1 + Y)\right]$$

Since $\frac{\partial I}{\partial X_1} = 0, \frac{\partial Y}{\partial X_1} = 0$

$$\Rightarrow \frac{\partial \dot{X}_1}{\partial X_1} = -k_3$$

And,

$$\frac{\partial \dot{Y}}{\partial X_1} = k_2 \frac{\partial}{\partial X_1}\left[N_Y I - \left(X_1 + Y\right)\right]$$

Since $\frac{\partial X_1}{\partial X_1} = 0, \frac{\partial Y}{\partial X_1} = 0$

$$\Rightarrow \frac{\partial \dot{Y}}{\partial X_1} = -k_2$$

$$\therefore \dot{m}_{2X_1} = \left(r + k_3\right)m_{2X_1} + \left[g_X k_3 + \left(g_Y + m_{2Y}\right)k_2\right] \qquad \text{(A.2.7)}$$

Similarly m_{2Y} is given by the multiplier equation:

$$\dot{m}_{2Y} = rm_{2Y} - \frac{\partial H_2}{\partial Y}$$

Now,

$$\frac{\partial H_2}{\partial Y} = \frac{\partial}{\partial Y}\left[-A + \left(g_X + m_{2X_1}\right)\dot{X}_1 + \left(g_X + m_{2X_2}\right)\dot{X}_2 + \left(g_Y + m_{2Y}\right)\dot{Y} + m_{2I}\dot{I}\right]$$

$$= \left(g_X + m_{2X_1}\right)\frac{\partial \dot{X}_1}{\partial Y} + \left(g_X + m_{2X_2}\right)\frac{\partial \dot{X}_2}{\partial Y} + \left(g_Y + m_{2Y}\right)\frac{\partial \dot{Y}}{\partial Y} + m_{2I}\frac{\partial \dot{I}}{\partial Y}$$

Since $\frac{\partial \dot{X}_2}{\partial Y} = 0, \frac{\partial \dot{I}}{\partial Y} = 0$.

$$\therefore \frac{\partial H_2}{\partial Y} = \left(r - \frac{\partial \dot{Y}}{\partial Y}\right)m_{2Y} - \left[g_Y \frac{\partial \dot{Y}}{\partial Y} + \left(g_X + m_{2X_1}\right)\frac{\partial \dot{X}_1}{\partial Y}\right]$$

But,

$$\frac{\partial \dot{X}_1}{\partial Y} = k_3 \frac{\partial}{\partial Y}\left[N_Y I - \left(X_1 + Y\right)\right]$$

Since $\frac{\partial X_1}{\partial Y} = 0, \frac{\partial I}{\partial Y} = 0$

$$\therefore \frac{\partial \dot{X}_1}{\partial Y} = -k_3$$

And,

$$\frac{\partial \dot{Y}}{\partial Y} = k_2 \frac{\partial}{\partial Y}\left[N_Y I - \left(X_1 + Y\right)\right]$$

Since $\frac{\partial X_1}{\partial Y} = 0, \frac{\partial I}{\partial Y} = 0$

$$\therefore \frac{\partial \dot{Y}}{\partial Y} = -k_2$$

$$\therefore \dot{m}_{2Y} = (r + k_2)m_{2Y} + \left[g_Y k_2 + (g_X + m_{2X_1})k_3\right] \quad \text{(A.2.8)}$$

From Equation (A.2.7) and (A.2.8),

$$\dot{m}_{2X_1} - \dot{m}_{2Y} = r(m_{2X_1} - m_{2Y})$$

Integrating both sides,

$$\int \frac{d(m_{2X_1} - m_{2Y})}{m_{2X_1} - m_{2Y}} = \int r dt + const_1$$

$$\Rightarrow \left|m_{2X_1} - m_{2Y}\right| = const_2 e^{rt}$$

Since $m_{2X_1}(T) = 0$ and $m_{2Y}(T) = 0$

$$\therefore const_2 e^{rt} = 0$$

$$\Rightarrow const_2 = 0$$

Hence $m_{2X_1} = m_{2Y}$ (A.2.9)

From Equation (A.2.8) and (A.2.9),

$$\dot{m}_{2Y} = (r + k_2)m_{2Y} + \left[g_Y k_2 + (g_X + m_{2Y})k_3\right]$$
$$= (r + k_2 + k_3)m_{2Y} + (g_Y k_2 + g_X k_3)$$

On solving,

$$\left[m_{2Y} e^{-(r+k_2+k_3)\tau}\right]_{\tau=t}^{\tau=T} = (g_Y k_2 + g_X k_3)\int_t^T e^{-(r+k_2+k_3)\tau} d\tau$$

Since $m_{2Y}(T) = 0$.

On simplifying,

$$m_{2Y} = \frac{g_Y k_2 + g_X k_3}{r + k_2 + k_3}\left[e^{-(r+k_2+k_3)(T-t)} - 1\right]$$

Since $m_{2X_1} = m_{2Y}$

Hence,

$$m_{2X_1} = \frac{g_Y k_2 + g_X k_3}{r + k_2 + k_3}\left[e^{-(r+k_2+k_3)(T-t)} - 1\right]$$

And m_{2X2} is given by the multiplier equation:

$$\dot{m}_{2X_2} = rm_{2X_2} - \frac{\partial H_2}{\partial X_2}$$

Now,

$$\frac{\partial H_2}{\partial X_2} = \frac{\partial}{\partial X_2}\left[-A + (g_X + m_{2X_1})\dot{X}_1 + (g_X + m_{2X_2})\dot{X}_2 + (g_Y + m_{2Y})\dot{Y} + m_{2I}\dot{I}\right]$$

$$= (g_X + m_{2X_1})\frac{\partial \dot{X}_1}{\partial X_2} + (g_X + m_{2X_2})\frac{\partial \dot{X}_2}{\partial X_2} + (g_Y + m_{2Y})\frac{\partial \dot{Y}}{\partial X_2} + m_{2I}\frac{\partial \dot{I}}{\partial X_2}$$

Since $\frac{\partial \dot{X}_1}{\partial X_2} = 0, \frac{\partial \dot{I}}{\partial X_2} = 0, \frac{\partial \dot{Y}}{\partial X_2} = 0$.

$$\therefore \frac{\partial H_2}{\partial X_2} = \left(r - \frac{\partial \dot{X}_2}{\partial X_2}\right)m_{2X_2} - \left[g_X \frac{\partial \dot{X}_2}{\partial X_2}\right]$$

But,

$$\frac{\partial \dot{X}_2}{\partial X_2} = k_4 \frac{\partial}{\partial X_2}\left[(N_X - N_Y)I - X_2\right]$$

Since $\frac{\partial I}{\partial X_2} = 0$.

$$\Rightarrow \frac{\partial \dot{X}_2}{\partial X_2} = -k_4$$

$$\therefore \dot{m}_{2X_2} = (r + k_4)m_{2X_2} + g_X k_4$$

On solving,

$$\left[m_{2X_2} e^{-(r+k_4)\tau}\right]_{\tau=t}^{\tau=T} = g_X k_4 \int_t^T e^{-(r+k_4)\tau} d\tau$$

Since $m_{2X_2}(T) = 0$

$$\therefore m_{2X_2} = \frac{g_X k_4}{r + k_4}\left[e^{-(r+k_4)(T-t)} - 1\right]$$

Appendix B

Table 1.1: Results of numerical analysis for 6.1(a)

A1	A2	A3	A4		I	Y	X_1	X_2	$X=X_1+X_2$	X+Y	π
0	25	25	0		0.362	1.689	2.952	18.234	21.186	22.875	21.195
25	0	25	0		0.367	2.096	2.688	18.614	21.301	23.397	19.785
25	25	0	0		0.295	1.889	2.055	15.294	17.349	19.237	19.312
25	25	25	0		0.422	2.476	3.228	22.235	25.463	27.939	19.115
0	0	25	0		0.278	1.168	2.165	13.086	15.252	16.419	16.640
0	25	25	25		0.489	1.969	3.511	21.492	25.003	26.972	16.334
0	25	0	0		0.186	0.928	1.429	9.236	10.666	11.593	15.977
25	25	0	25		0.442	2.209	2.696	19.030	21.726	23.935	15.381
25	0	0	0		0.197	1.352	1.200	9.823	11.024	12.376	15.103
25	0	25	25		0.493	2.372	3.240	21.829	25.069	27.441	14.841
0	25	0	25		0.367	1.315	2.203	13.746	15.949	17.264	13.545
25	25	25	25		0.532	2.718	3.713	25.060	28.773	31.491	13.409
0	0	25	25		0.430	1.499	2.828	16.944	19.772	21.271	12.946
25	0	0	25		0.375	1.733	1.961	14.255	16.216	17.949	12.521
0	0	0	0		0.021	0.141	0.112	0.968	1.080	1.220	3.553
0	0	0	25		0.255	0.629	1.087	6.649	7.737	8.366	3.382

The optimal advertising policy is given by the shaded row in the Table shown above, that is, $(0, 25, 25, 0)$.

Table 1.2: Results of numerical analysis for 6.1(b)

A1	A2	A3	A4		I	Y	X_1	X_2	$X=X_1+X_2$	X+Y	π
25	25	25	0		0.747	3.606	5.284	34.707	39.991	43.597	218.543
25	25	0	0		0.621	3.036	4.144	27.965	32.109	35.145	207.461
25	0	25	0		0.666	3.107	4.509	29.692	34.202	37.309	198.077
25	25	25	25		0.809	3.753	5.579	36.424	42.003	45.756	184.624
25	25	0	25		0.717	3.256	4.584	30.529	35.112	38.368	181.516
0	25	25	0		0.628	2.494	4.517	27.504	32.021	34.515	176.675
25	0	25	25		0.750	3.301	4.897	31.954	36.852	40.153	169.286
25	0	0	0		0.476	2.314	2.923	20.326	23.248	25.563	162.949
0	25	25	25		0.722	2.709	4.948	30.017	34.965	37.675	150.254
25	0	0	25		0.613	2.617	3.529	23.859	27.388	30.006	146.188
0	25	0	0		0.404	1.590	2.710	16.843	19.553	21.143	129.636
0	0	25	0		0.472	1.695	3.187	19.134	22.321	24.016	125.543
0	25	0	25		0.563	1.935	3.399	20.854	24.253	26.187	117.418
0	0	25	25		0.611	2.000	3.797	22.688	26.486	28.486	108.984
0	0	0	25		0.334	0.818	1.433	8.731	10.163	10.981	32.217
0	0	0	0		0.066	0.283	0.363	2.503	2.866	3.149	23.230

The optimal advertising policy is given by the shaded row in the Table shown above, that is, **(25, 25, 25, 0)**.

Table 2.1: Results of numerical analysis for 6.2(a)

A1	A2	A3	A4		I	Y	X_1	X_2	$X=X_1+X_2$	X+Y	π
50	50	0	0		0.670	3.380	4.516	30.746	35.262	38.642	183.010
50	0	0	0		0.523	2.615	3.242	22.726	25.967	28.582	160.104
50	0	50	0		0.716	3.450	4.911	32.588	37.499	40.949	149.609
50	50	50	0		0.792	3.957	5.669	37.569	43.238	47.195	144.741
0	50	50	0		0.681	2.756	4.988	30.396	35.384	38.140	127.903
0	50	0	0		0.449	1.791	3.059	19.006	22.065	23.857	124.377
50	50	0	50		0.766	3.607	4.970	33.393	38.362	41.970	107.808
0	0	50	0		0.524	1.900	3.596	21.548	25.144	27.044	97.522
50	0	0	50		0.668	2.942	3.894	26.528	30.422	33.364	95.821
50	0	50	50		0.798	3.645	5.302	34.865	40.167	43.812	70.931
0	50	0	50		0.619	2.168	3.812	23.390	27.202	29.370	65.612
50	50	50	50		0.851	4.101	5.956	39.242	45.197	49.298	60.379
0	50	50	50		0.774	2.976	5.428	32.955	38.382	41.358	51.876
0	0	50	50		0.668	2.226	4.248	25.345	29.593	31.819	33.193
0	0	0	0		0.066	0.283	0.363	2.503	2.866	3.149	23.230
0	0	0	50		0.377	0.913	1.623	9.839	11.462	12.375	-7.350

The optimal advertising policy is given by the shaded row in the Table shown above, that is, **(50, 50, 0, 0)**.

Table 2.2: Results of numerical analysis for 6.2(b)

A1	A2	A3	A4		I	Y	X_1	X_2	$X=X_1+X_2$	X+Y	π
10	10	10	0		0.667	3.061	4.617	29.982	34.599	37.659	235.334
10	10	10	10		0.731	3.207	4.910	31.684	36.594	39.800	231.310
10	0	10	10		0.663	2.775	4.219	27.261	31.480	34.254	201.299
10	0	10	0		0.581	2.591	3.852	25.124	28.976	31.567	201.218
10	10	0	10		0.631	2.723	3.942	25.964	29.906	32.629	199.606
10	10	0	0		0.540	2.522	3.540	23.618	27.158	29.680	197.549
0	10	10	10		0.633	2.293	4.184	25.372	29.556	31.848	183.041
0	10	10	0		0.543	2.092	3.783	23.039	26.822	28.914	181.099
10	0	0	10		0.525	2.146	2.961	19.827	22.788	24.934	153.948
10	0	0	0		0.402	1.885	2.438	16.784	19.223	21.107	145.257
0	0	10	10		0.519	1.664	3.124	18.725	21.849	23.513	132.765
0	10	0	10		0.475	1.595	2.789	17.132	19.920	21.515	128.296
0	0	10	0		0.395	1.399	2.595	15.644	18.240	19.639	123.717
0	10	0	0		0.337	1.305	2.209	13.757	15.966	17.272	116.447
0	0	0	10		0.273	0.690	1.176	7.234	8.409	9.099	48.089
0	0	0	0		0.066	0.283	0.363	2.503	2.866	3.149	23.230

The optimal advertising policy is given by the shaded row in the Table shown above, that is, **(10, 10, 10, 0)**.

Table 3.1: Results of numerical analysis for 6.3(a)

A1	A2	A3	A4		I	Y	X_1	X_2	$X=X_1+X_2$	X+Y	π
25	25	25	0		0.747	3.606	5.284	34.707	39.991	43.597	218.543
25	25	0	0		0.621	3.036	4.144	27.965	32.109	35.145	207.461
25	0	25	0		0.666	3.107	4.509	29.692	34.202	37.309	198.077
25	25	25	25		0.809	3.753	5.579	36.424	42.003	45.756	184.624
25	25	0	25		0.717	3.256	4.584	30.529	35.112	38.368	181.516
0	25	25	0		0.628	2.494	4.517	27.504	32.021	34.515	176.675
25	0	25	25		0.750	3.301	4.897	31.954	36.852	40.153	169.286
25	0	0	0		0.476	2.314	2.923	20.326	23.248	25.563	162.949
0	25	25	25		0.722	2.709	4.948	30.017	34.965	37.675	150.254
25	0	0	25		0.613	2.617	3.529	23.859	27.388	30.006	146.188
0	25	0	0		0.404	1.590	2.710	16.843	19.553	21.143	129.636
0	0	25	0		0.472	1.695	3.187	19.134	22.321	24.016	125.543
0	25	0	25		0.563	1.935	3.399	20.854	24.253	26.187	117.418
0	0	25	25		0.611	2.000	3.797	22.688	26.486	28.486	108.984
0	0	0	25		0.334	0.818	1.433	8.731	10.163	10.981	32.217
0	0	0	0		0.066	0.283	0.363	2.503	2.866	3.149	23.230

The optimal advertising policy is given by the shaded row in the Table shown above, that is, **(25, 25, 25, 0)**.

Table 3.2: Results of numerical analysis for 6.3(b)

A1	A2	A3		I	Y	X1	X2	X=X1+X2	X+Y	π
25	25	0		0.460	2.127	2.326	16.273	18.599	20.726	104.287
25	25	25		0.602	2.439	2.951	19.912	22.863	25.302	88.528
25	0	0		0.321	1.591	1.477	10.988	12.465	14.056	80.203
25	0	25		0.505	1.983	2.260	15.548	17.807	19.790	73.210
0	25	0		0.261	0.932	1.394	8.808	10.203	11.135	57.461
0	25	25		0.464	1.358	2.245	13.764	16.009	17.367	54.246
0	0	25		0.313	0.728	1.253	7.576	8.828	9.556	21.954
0	0	0		0.036	0.176	0.149	1.150	1.299	1.475	11.063

The optimal advertising policy is given by the shaded row in the Table shown above, that is, **(25, 25, 0)**.

Table 4.1: Results of numerical analysis for 6.4(a)

A1	A2	A3	A4	I	Y	X_1	X_2	$X=X_1+X_2$	X+Y	π
25	25	25	0	0.747	10.106	18.081	14.461	32.542	42.648	299.434
25	25	0	0	0.621	9.040	13.818	11.652	25.470	34.510	270.930
25	25	25	25	0.809	10.366	19.121	15.177	34.298	44.664	269.477
25	0	25	0	0.666	8.374	15.619	12.372	27.991	36.364	266.846
25	25	0	25	0.717	9.428	15.369	12.720	28.089	37.517	250.891
0	25	25	0	0.628	5.847	16.039	11.460	27.499	33.346	244.291
25	0	25	25	0.750	8.716	16.988	13.314	30.302	39.018	243.268
0	25	25	25	0.722	6.227	17.559	12.507	30.067	36.294	223.661
25	0	0	0	0.476	6.899	9.719	8.469	18.188	25.087	207.752
25	0	0	25	0.613	7.433	11.855	9.941	21.796	29.229	199.124
0	0	25	0	0.472	3.383	11.628	7.972	19.601	22.984	172.725
0	25	0	0	0.404	4.171	9.335	7.018	16.353	20.524	170.089
0	25	0	25	0.563	4.777	11.759	8.689	20.448	25.225	167.100
0	0	25	25	0.611	3.920	13.777	9.453	23.230	27.150	164.347
0	0	0	25	0.334	1.724	4.994	3.638	8.632	10.356	51.995
0	0	0	0	0.066	0.785	1.238	1.043	2.281	3.066	28.698

The optimal advertising policy is given by the shaded row in the Table shown above, that is, **(25, 25, 25, 0)**.

Table 4.2: Results of numerical analysis for 6.4(b)

A1	A2	A3	A4	I	Y	X_1	X_2	$X=X_1+X_2$	X+Y	π
25	25	0	0	0.621	9.040	13.818	11.652	25.470	34.510	169.410
25	25	25	0	0.747	10.106	18.081	14.461	32.542	42.648	165.621
25	0	25	0	0.666	8.374	15.619	12.372	27.991	36.364	150.571
25	25	0	25	0.717	9.428	15.369	12.720	28.089	37.517	137.004
25	0	0	0	0.476	6.899	9.719	8.469	18.188	25.087	136.368
25	25	25	25	0.809	10.366	19.121	15.177	34.298	44.664	127.391
0	25	25	0	0.628	5.847	16.039	11.460	27.499	33.346	122.879
25	0	25	25	0.750	8.716	16.988	13.314	30.302	39.018	116.088
25	0	0	25	0.613	7.433	11.855	9.941	21.796	29.229	110.662
0	25	0	0	0.404	4.171	9.335	7.018	16.353	20.524	99.792
0	25	25	25	0.722	6.227	17.559	12.507	30.067	36.294	90.126
0	0	25	0	0.472	3.383	11.628	7.972	19.601	22.984	83.550
0	25	0	25	0.563	4.777	11.759	8.689	20.448	25.225	77.396
0	0	25	25	0.611	3.920	13.777	9.453	23.230	27.150	57.990
0	0	0	0	0.066	0.785	1.238	1.043	2.281	3.066	19.414
0	0	0	25	0.334	1.724	4.994	3.638	8.632	10.356	12.448

The optimal advertising policy is given by the shaded row in the Table shown above, that is, **(25, 25, 0, 0)**.

Table 4.3: Results of numerical analysis for 6.4(c)

A1	A2	A3	A4	I	Y	X_1	X_2	$X=X_1+X_2$	X+Y	π
25	25	25	0	0.747	10.106	18.081	14.461	32.542	42.648	231.313
25	25	0	0	0.621	9.040	13.818	11.652	25.470	34.510	220.900
25	0	25	0	0.666	8.374	15.619	12.372	27.991	36.364	207.170
25	25	25	25	0.809	10.366	19.121	15.177	34.298	44.664	196.715
25	25	0	25	0.717	9.428	15.369	12.720	28.089	37.517	193.924
0	25	25	0	0.628	5.847	16.039	11.460	27.499	33.346	178.502
25	0	25	25	0.750	8.716	16.988	13.314	30.302	39.018	177.474
25	0	0	0	0.476	6.899	9.719	8.469	18.188	25.087	173.228
25	0	0	25	0.613	7.433	11.855	9.941	21.796	29.229	155.019
0	25	25	25	0.722	6.227	17.559	12.507	30.067	36.294	151.072
0	25	0	0	0.404	4.171	9.335	7.018	16.353	20.524	132.941
0	0	25	0	0.472	3.383	11.628	7.972	19.601	22.984	123.316
0	25	0	25	0.563	4.777	11.759	8.689	20.448	25.225	119.064
0	0	25	25	0.611	3.920	13.777	9.453	23.230	27.150	105.299
0	0	0	25	0.334	1.724	4.994	3.638	8.632	10.356	30.381
0	0	0	0	0.066	0.785	1.238	1.043	2.281	3.066	24.063

The optimal advertising policy is given by the shaded row in the Table shown above, that is, **(25, 25, 25, 0)**.

Table 5.1: Results of numerical analysis for 6.5(a)

A1	A2	A3	A4	I	Y	X_1	X_2	$X=X_1+X_2$	X+Y	π
25	25	0	0	0.621	9.040	13.818	11.652	25.470	34.510	169.410
25	25	25	0	0.747	10.106	18.081	14.461	32.542	42.648	165.621
25	0	25	0	0.666	8.374	15.619	12.372	27.991	36.364	150.571
25	25	0	25	0.717	9.428	15.369	12.720	28.089	37.517	137.004
25	0	0	0	0.476	6.899	9.719	8.469	18.188	25.087	136.368
25	25	25	25	0.809	10.366	19.121	15.177	34.298	44.664	127.391
0	25	25	0	0.628	5.847	16.039	11.460	27.499	33.346	122.879
25	0	25	25	0.750	8.716	16.988	13.314	30.302	39.018	116.088
25	0	0	25	0.613	7.433	11.855	9.941	21.796	29.229	110.662
0	25	0	0	0.404	4.171	9.335	7.018	16.353	20.524	99.792
0	25	25	25	0.722	6.227	17.559	12.507	30.067	36.294	90.126
0	0	25	0	0.472	3.383	11.628	7.972	19.601	22.984	83.550
0	25	0	25	0.563	4.777	11.759	8.689	20.448	25.225	77.396
0	0	25	25	0.611	3.920	13.777	9.453	23.230	27.150	57.990
0	0	0	0	0.066	0.785	1.238	1.043	2.281	3.066	19.414
0	0	0	25	0.334	1.724	4.994	3.638	8.632	10.356	12.448

The optimal advertising policy is given by the shaded row in the Table shown above, that is, **(25, 25, 0, 0)**.

Table 5.2: Results of numerical analysis for 6.5(b)

A1	A2	A3	A4		I	Y	X_1	X_2	$X=X_1+X_2$	X+Y	π
25	25	25	0		0.747	22.156	11.209	14.461	25.670	47.827	251.979
25	25	0	0		0.621	18.313	8.647	11.652	20.299	38.612	236.691
25	0	25	0		0.666	19.104	9.849	12.372	22.220	41.324	229.364
25	25	25	25		0.809	23.347	12.003	15.177	27.180	50.527	222.003
25	25	0	25		0.717	20.099	9.838	12.720	22.558	42.656	216.762
25	0	25	25		0.750	20.677	10.897	13.314	24.212	44.888	205.841
0	25	25	0		0.628	16.851	10.106	11.460	21.566	38.417	203.610
25	0	0	0		0.476	13.660	6.220	8.469	14.689	28.349	186.973
0	25	25	25		0.722	18.601	11.272	12.507	23.780	42.381	183.081
25	0	0	25		0.613	16.134	7.869	9.941	17.810	33.944	178.699
0	25	0	0		0.404	10.603	5.940	7.018	12.958	23.561	147.595
0	0	25	0		0.472	11.755	7.528	7.972	15.500	27.255	147.215
0	25	0	25		0.563	13.418	7.816	8.689	16.506	29.923	145.114
0	0	25	25		0.611	14.244	9.187	9.453	18.640	32.884	139.197
0	0	0	25		0.334	6.109	3.764	3.638	7.402	13.511	51.375
0	0	0	0		0.066	1.697	0.822	1.043	1.865	3.563	26.578

The optimal advertising policy is given by the shaded row in the Table shown above, that is, **(25, 25, 25, 0)**.